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A Sandwich Standard Error Estimator for Exploratory Factor Analysis With Nonnormal Data and Imperfect Models

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Abstract

This article is concerned with standard errors (*SE*s) and confidence intervals (CIs) for exploratory factor analysis (EFA) in different situations. The authors adapt a sandwich *SE* estimator for EFA parameters to accommodate nonnormal data and imperfect models, factor extraction with maximum likelihood and ordinary least squares, and factor rotation with CF-varimax, CF-quartimax, geomin, or target rotation. They illustrate the sandwich *SE*s and CIs using nonnormal continuous data and ordinal data. They also compare *SE* estimates and CIs of the conventional information method, the sandwich method, and the bootstrap method using simulated data. The sandwich method and the bootstrap method are more satisfactory than the information method for EFA with nonnormal data and model approximation error.

Keywords

factor analysis, latent variable models, factor rotation, standard errors

Introduction

Exploratory factor analysis (EFA) is a widely used statistical procedure in the social and behavioral sciences. It is a data-driven approach for understanding correlations among manifest variables with fewer latent factors. Cudeck and O'Dell (1994) encouraged factor analysts to examine both point estimates and standard error (*SE*) estimates (and confidence intervals [CIs]) when interpreting EFA results. If factor analysts examine only point estimates, results and conclusions can be misleading because these point estimates may be associated with *SE*s of different sizes or CIs of different widths.

EFA *SE*s were originally derived for maximum likelihood (ML) estimates under the assumptions that (a) the EFA model fits perfectly in the population and (b) manifest variables are

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normally distributed (Jennrich, 1973). Because the *SE* involves the information matrix, the authors refer to it as the information *SE*.[1] The two assumptions for the information *SE* do not hold in many EFA applications.

The authors present a sandwich method for estimating *SE*s for rotated factor loadings and factor correlations in EFA. They first describe a general form of the sandwich *SE* estimator and then adapt it to accommodate different features of EFA models: three data distributions (normal data, nonnormal continuous data, and ordinal data), two estimation methods (ML and ordinary least squares [OLS]), four rotation criteria (CF-varimax, CF-quartimax, geomin, and target), and imperfect models. A sandwich *SE* estimator was first developed for estimating *SE*s in nonlinear regression with model error (White, 1981); it has been adapted to estimate EFA *SE*s with nonnormal variables and/or model error (Asparouhov & Muthén, 2009; Lee, Zhang, & Edwards, 2012; Yuan, Marshall, & Bentler, 2002). Previous adaptations focused on deriving EFA *SE*s for a particular combination of the estimation method and data distribution; the authors' goal is to fully exploit the versatility of the sandwich *SE* estimator in EFA. The authors explain how the current adaptation includes previous adaptations as special cases. In addition, they implement the sandwich *SE* estimator in an R package EFAutilities (Zhang, Jiang, Hattori, & Trichtinger, 2018) to make it accessible to applied researchers.

The rest of the article is organized as follows. The authors first describe the estimation and interpretation of EFA. They then present the concept of imperfect EFA models and their consequences in model estimation and model interpretation. They next describe a sandwich *SE* estimator, which provides appropriate *SE*s for imperfect EFA models in a variety of conditions. In particular, they explain how components of the sandwich *SE* estimator are changed to accommodate different features of the EFA model. They demonstrate the versatility of the sandwich *SE* estimator with two empirical data sets and explore its statistical properties with simulated data. Finally, they provide several remarks on its theoretical and practical implications.

The Estimation and Interpretation of EFA

The EFA model is often estimated using a two-step procedure. The first step is factor extraction, in which an unrotated factor loading matrix $\mathbf{A}$ is obtained by minimizing a discrepancy function of the sample correlation matrix $\mathbf{R}$ and the model implied correlation matrix $\mathbf{P} = \mathbf{A}\mathbf{A}' + \mathbf{D}_\psi$. The $p \times p$ diagonal matrix $\mathbf{D}_\psi$ contains unique variances. Two widely used discrepancy functions are ML and OLS. Except in one-factor models, the unrotated factor loading matrix $\mathbf{A}$ is rarely interpretable. The second step of EFA is to rotate $\mathbf{A}$ with the aim of improving its interpretability. One can conduct factor rotation obliquely or orthogonally. Factors are allowed to be correlated in oblique rotation, but they are uncorrelated in orthogonal rotation. Oblique rotation tends to produce clearer factor loading matrices than orthogonal rotation. Examples of factor rotation methods are CF-varimax, CF-quartimax, geomin, and target rotation (Browne, 2001).

The authors propose to interpret factor loadings with the aid of their CIs. We can divide factor loadings into four types according to their CIs: a strongly salient loading, a salient loading, a small factor loading, and a noninformative factor loading. Let a be a criterion value (for example, 0.3 or 0.4) chosen by a factor analyst according to her substantive knowledge. If the lower end of the CI of a positive loading is larger than a (or the upper end of the CI of a negative salient loading is less than $-a$), the factor loading is a strongly salient loading. A strongly salient loading indicates a manifest variable that defines the corresponding factor. If the CI includes a (or $-a$) but does not include zero, the factor loading is a salient loading. A salient loading indicates some relation between the manifest variable and the factor, but the relation tends to be weaker than that of a strongly salient loading. If the whole CI is between $-a$ and a, the factor loading is a small loading. A small loading indicates that a factor does not importantly influence the corresponding manifest variable. In particular, if the CI includes zero, it indicates the relation between the factor and the manifest variable is not detectable at the current sample size. If the CI contains both zero and a (or $-a$), the factor loading is a noninformative loading. A noninformative loading provides little information about the strength or the direction of the relation between the manifest variable and the factor. Of course, the criterion value a is still subjectively chosen. Because EFA is an exploratory procedure and it should be followed by a confirmatory procedure, sound human judgment often aids rather than hinders the extraction of information from data using EFA. Nevertheless, the authors feel that the use of CIs will improve decisions regarding interpreting loadings regardless of how the criterion value a is chosen. For example, let two factor loadings λ_1 and λ_2 have the same point estimate of 0.4, but the CI for λ_1 is [0.35, 0.45] and the CI for λ_2 is [–0.21, 1.0]. Let the criterion value a be 0.3. According to the CIs, λ_1 is a strongly salient factor loading, but λ_2 is a noninformative loading. One would regard both factor loadings as salient if one does not consider CIs.

Imperfect EFA Models

A parsimonious model can never capture the full richness of real-world phenomena. The best researchers can hope for is that a model ''approximately'' holds in the population. MacCallum (2003) examined consequences of imperfect EFA models. He argued that imperfect EFA models are unavoidable; for example, the influence of factors on manifest variables could be nonlinear, or there could be too many minor factors to be included in the model.

The unavoidability of imperfect EFA models has profound implications for the estimation and interpretation of EFA models. Let $\mathbf{P}_0$ be the population correlation matrix. Factor analyzing $\mathbf{P}_0$ does not produce a perfectly fitting EFA model. Nevertheless, minimizing a discrepancy function $f(\mathbf{P}_0, \mathbf{P}(\boldsymbol{\theta}))$ with regard to $\boldsymbol{\theta}$ produces a set of parameter values $\boldsymbol{\theta}_0$, which includes the factor loading matrix $\boldsymbol{\Lambda}_0$ and the factor correlation matrix $\boldsymbol{\Phi}_0$. The minimum discrepancy function value $f(\mathbf{P}_0, \mathbf{P}(\boldsymbol{\theta}_0))$ is referred to as the error of approximation. Although the EFA model with $\boldsymbol{\theta}_0$ does not account for $\mathbf{P}_0$ completely, the EFA model can still be useful if it helps us understand $\mathbf{P}_0$ with m common factors. Of course, the EFA model is not helpful if the error of approximation is large. A commonly used method for measuring the error of the approximation is the root mean square error of approximation (RMSEA), and RMSEA values of 0.00, 0.05, 0.08, and 0.10 correspond to perfect fit, close fit, acceptable fit, and unacceptable fit, respectively (Browne & Cudeck, 1993).

Note that $\boldsymbol{\theta}_0$ contains population values rather than sample estimates because the EFA model is estimated with the population correlation matrix $\mathbf{P}_0$. Let $\mathbf{R}$ be a sample correlation matrix drawn from the population. Factor analyzing $\mathbf{R}$ produces $\widehat{\boldsymbol{\theta}}$, which includes $\widehat{\boldsymbol{\Lambda}}$ and $\widehat{\boldsymbol{\Phi}}$. The sample estimate $\widehat{\boldsymbol{\theta}}$ is a consistent estimate for the population values $\boldsymbol{\theta}_0$ even with model approximation error and nonnormal data.

Estimating *SE*s for $\widehat{\boldsymbol{\theta}}$ with the information method assumes that there is no model error in the population. The authors next describe a sandwich *SE* method, which provides consistent *SE* estimates for $\widehat{\boldsymbol{\theta}}$ even with model approximation error. In addition, it can be adapted to accommodate different estimation methods, different factor rotation methods, and different data distributions.

A Sandwich Method for Estimating *SE*s in EFA

The large sample distribution of $\widehat{\boldsymbol{\theta}}$ is multivariate normal with mean vector $\boldsymbol{\theta}_0$ and covariance matrix $\boldsymbol{\Omega}$. The covariance matrix $\boldsymbol{\Omega}$ is computed using a sandwich method[2]

$$\mathbf{\Omega} = \mathbb{A}^{11}\left(\frac{\partial^2 f}{\partial\theta\partial\rho'}\mathbf{\Gamma}\frac{\partial^2 f}{\partial\rho\partial\theta'}\right)\mathbb{A}^{11}. \tag{1}$$

The middle part of the sandwich method involves two terms: $\mathbf{\Gamma}$ is the asymptotic covariance matrix of the vector of sample correlations; $\frac{\partial^2 f}{\partial\theta\partial\rho'}$ is a matrix of the partial derivatives of the discrepancy function with regard to model parameters and manifest variable correlations. The outer parts of the sandwich method are both $\mathbb{A}^{11}$, which is obtained from a matrix inversion

$$\begin{bmatrix} \mathbb{A}^{11} & \mathbb{A}^{12} \\ \mathbb{A}^{21} & \mathbb{A}^{22} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 f}{\partial\theta\partial\theta'} & \frac{\partial c(\boldsymbol{\theta})'}{\partial\boldsymbol{\theta}} \\ \frac{\partial c(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}'} & 0 \end{bmatrix}^{-1}. \tag{2}$$

The matrix $(\partial^2 f)/(\partial\theta\partial\theta')$ contains the second derivatives of the discrepancy function with regard to model parameters; the vector $c(\boldsymbol{\theta})$ contains constraint functions imposed on rotated factor loadings and factor correlations to deal with different factor rotation methods.

Although similar *SE* estimates have been described previously for EFA with particular combinations of data types, estimation methods, and rotation methods (Yuan et al., 2002, for ML estimates, orthogonal rotation, and continuous data; Lee et al., 2012, for OLS estimates, oblique rotation, and ordinal variables; Asparouhov & Muthén, 2009, for exploratory structural equation models), the authors' goal here is to exploit the versatility of the sandwich *SE* estimator to its full extent and to show how to adapt it to accommodate different features of EFA models.

Nonnormal Data

We can readily adapt the sandwich *SE* estimator to accommodate nonnormal data because we factor analyze the correlation matrix rather than scaling the results after factor analyzing the covariance matrix. Note that factor analyzing the correlation matrix is much more common than factor analyzing the covariance matrix in EFA. The key adaptation is to properly specify the asymptotic covariance matrix ($\mathbf{\Gamma}$ in Equation 1) of manifest variable correlations for different types of data. Browne and Shapiro (1986) derived a matrix expression of $\mathbf{\Gamma}$ for continuous but nonnormal data. When data are ordinal, their polychoric correlations are factor analyzed. The polychoric correlations are estimated using a two-stage method (Olsson, 1979), and their asymptotic covariance matrix $\mathbf{\Gamma}$ is estimated using an estimating equation method (Yuan & Schuster, 2013).

Different Levels of Model Approximation Error

The sandwich *SE* estimator allows any level of model approximation error. When no model approximation error is present in the population, we can simplify the sandwich *SE* estimator in two ways. First, the second derivatives $(\partial^2 f)/(\partial\theta\partial\theta')$ in Equation 2 are no longer necessary. We can replace the matrix of second derivatives by the product of first derivatives $(\partial f/\partial\theta)(\partial f/\partial\theta)'$. Second, $\mathbf{\Gamma}$ can be estimated using the model-implied correlations $\widehat{\mathbf{P}}$ instead of $\mathbf{R}$.

We can greatly simplify the *SE* estimation in EFA if no model approximation error combines with normal data and ML estimation. The submatrix $\mathbb{A}^{11}$ in Equation 2 alone provides *SE*s (Jennrich, 1974); they are information *SE*s. The information *SE*s are commonly computed in EFA software (CEFA, Browne, Cudeck, Tateneni, & Mels, 2010; PROC FACTOR, SAS Institute, 2006).

ML Estimation and OLS Estimation

Different factor estimation methods require modifications to the partial derivatives $(\partial^2 f)/(\partial\theta\partial\rho')$ in the middle part and the second derivatives $(\partial^2 f)/(\partial\theta\partial\theta')$ in the outer part of the sandwich *SE* estimator. Such derivatives for ML estimation and OLS estimation were described by Zhang, Preacher, and Jennrich (2012).

Factor Rotation Methods

Different rotation methods require modification to the constraint function $c(\boldsymbol{\theta})$ in Equation 2. The constraint functions corresponding to the Crawford-Ferguson family were derived by Jennrich (1973), and constraint functions corresponding to CF-varimax, CF-quartimax, geomin, and target rotation are documented in Tateneni (1998).

An approximate 95% CI for a rotated factor loading is constructed by $(\widehat{\lambda}_L, \widehat{\lambda}_U) = (\widehat{\lambda} - 1.96\times\widehat{\sigma}_\lambda, \widehat{\lambda} + 1.96\times\widehat{\sigma}_\lambda)$. Here, $\widehat{\lambda}$ is a point estimate and $\widehat{\sigma}_\lambda$ is its *SE* estimate, which is obtained by computing the square root of a diagonal element of $\boldsymbol{\Omega}$ in Equation 1.

Empirical Illustrations

To illustrate the versatility of the sandwich *SE* estimator of Equation 1, the authors compute *SE*s and CIs for two empirical data sets. They present point estimates and *SE* estimates to save space. They include the tables for CIs and the R code for empirical illustrations in an online supporting file.[3]

EFA With Nonnormal Continuous Variables

Luo et al. (2008) reported a study on marital satisfaction of urban Chinese couples. Their participants were 537 couples in the first 3 years of their first marriage. The current illustration includes 28 facet scores of the Chinese Personality Assessment Inventory (Cheung et al., 1996) from the 537 wives. The authors extracted four factors from the sample correlation matrix using ML. The 90% CI for the RMSEA is [0.038, 0.049], which indicates close fit for the four-factor EFA model (Browne & Cudeck, 1993). The test of perfect fit is rejected, however. The factor rotation method was oblique CF-varimax.

The authors estimate *SE*s for rotated factor loadings and factor correlations using three methods: the sandwich method, the bootstrap method, and the information method. The information method assumes normal variables and a perfect EFA model, but the sandwich method and the bootstrap method do not make such assumptions. The number of bootstrap samples was 2,000.

Table 1 reports point estimates and two types of *SE* estimates (sandwich *SE* estimates, bootstrap *SE* estimates). These two types of *SE* estimates agree with each other up to the second decimal place for most parameters; the largest difference is about 0.01.[4] According to the substantive theory, the authors expect some factor loadings to be large (shown in bold font) and other factor loadings to be small (shown in regular font). We can construct CIs using the point estimates and *SE* estimates to assess these expectations. CIs constructed with these two types of *SE* estimates are essentially the same. Let the criterion value be 0.3. Most of these expected large loadings are strongly salient loadings or salient loadings. The 95% CI for λ_{11} is [0.60, 0.75]; it is interpreted as a strongly salient loading because its lower end is larger than 0.3; it indicates a strong association between the manifest variable ''novelty'' and the factor ''social potency.'' The 95% CI for λ_{61} is [0.22, 0.47]; it is interpreted as a salient loading because it contains 0.3 but not 0; it indicates a weak to moderate level of association between the manifest variable "aesthetics" and the factor "social potency." Most expected small loadings are small. The CI for λ_{91} is [–0.08, 0.08]; it is interpreted as a small loading because the whole CI is between -0.3 and 0.3; it indicates a negligible association between the manifest variable "responsibility" and the factor "social potency." Several factor loadings deviate from the expected pattern, however. The variable "traditionalism-modernity" was expected to load highly on "interpersonal relatedness," but the CI is [–0.02, 0.17]; it is interpreted as a small loading. The loading of the same variable on "accommodation" was expected to be low, but the CI is [–0.72, –0.53]; it is interpreted as a strongly salient loading.

EFA With Ordinal Variables

The second empirical data set involves 228 participants and 44 ordinal variables (Luo, 2005).[5] These variables are items of the Big Five Inventory (John, Donahue, & Kentle, 1991). The variables are five-point Likert-type scales: *disagree strongly, disagree a little, neither agree nor disagree, agree a little*, and *agree strongly*. Because the data are ordinal variables, the polychoric correlation matrix is factor analyzed instead of the Pearson correlation matrix. The factor estimation method is OLS estimation. A five-factor model fits the data well but not perfectly: a 90% CI for the RMSEA is [0.043, 0.054]. Model error is present in the EFA model. The authors illustrate the sandwich *SE* estimates for four oblique rotation methods: CF-varimax, CF-quartimax, geomin, and target rotation. The sandwich method involves the nontrivial task of estimating the asymptotic covariance matrix ($\mathbf{\Gamma}$ in Equation 1) of polychoric correlations. The polychoric correlation matrix is of order 44 by 44; their asymptotic covariance matrix is of order $(44\times43)/2$ by $(44\times43)/2$; the number of nonduplicated elements in the matrix is 447,931.

The point estimates for rotated factor loadings and factor correlations are very close under the four rotation methods. The congruence coefficients (Gorsuch, 1983, p. 285) of "extraversion" among the four rotation methods range from 0.996 to 1.000; the congruence coefficient ranges for "agreeableness," "conscientiousness," "neuroticism," and "openness" are 0.987 to 1.000, 0.996 to 1.000, 0.991 to 1.000, and 0.999 to 1.000, respectively. Figure 1 displays the comparisons of *SE* estimates under the four rotation methods. *SE* estimates under CF-varimax, geomin, and target rotation are similar. *SE* estimates under CF-quartimax rotation differ from those of the other three rotation methods.

Table 2 presents point estimates and *SE* estimates with geomin rotation. Results of CF-varimax, CF-quartimax, and target rotation are presented in the online supporting file. According to the substantive theory, the authors expect some factor loadings to be large (shown in bold font) and other factor loadings to be small (shown in regular font). We can construct CIs using the point estimates and *SE* estimates to assess these expectations. Most of these expected large loadings are strongly salient loadings or salient loadings. The 95% CI for λ_{11} is [0.67, 0.90]; it is interpretd as a strongly salient loading because its lower end is larger than 0.3; it indicates a strong association between the manifest variable "talkative" and the factor "extraversion." Most expected small loadings are small. The CI for λ_{12} is [–0.09, 0.26]; it is interpreted as a small loading; it indicates a negligible association between the manifest variable "responsibility" and the factor "social potency." Several CIs are wide. The 95% CI of factor loading of "plans" on "agreeableness" is [–0.13, 0.39]; it is interpreted as noninformative because it includes both zero and 0.3; the inference on the loading is inconclusive due to the wide CI.

Table 1. Factor Analysis of Luo et al.'s (2008) Personality Data.

	Rotated factor loadings			
	Socplot	Depend	Accom	Interper
Novelty	**.67** [.04, .04]	.02 [.05, .05]	.12 [.04, .04]	.15 [.05, .06]
Diversity	**.53** [.06, .06]	.15 [.05, .05]	.19 [.04, .04]	.39 [.06, .07]
Diverse-Thinking	**.46** [.06, .06]	−.01 [.05, .05]	−.14 [.05, .05]	.31 [.07, .07]
Leadership	**.67** [.04, .04]	.04 [.04, .05]	−.3 [.05, .05]	−.08 [.06, .06]
Logical-affective	**.47** [.05, .05]	−.16 [.05, .05]	−.07 [.04, .04]	.18 [.06, .06]
Aesthetics	**.34** [.07, .07]	.09 [.05, .05]	−.13 [.05, .05]	.22 [.07, .07]
Extroversion-introversion	**.57** [.05, .05]	−.10 [.07, .07]	.00 [.04, .04]	−.07 [.05, .06]
Enterprise	**.54** [.06, .07]	−.43 [.06, .07]	.10 [.04, .04]	−.27 [.04, .05]
Responsibility	.00 [.04, .04]	−.62 [.05, .05]	−.06 [.04, .04]	.12 [.05, .05]
Emotionality	.03 [.04, .05]	**.75** [.04, .04]	.05 [.04, .04]	.06 [.04, .04]
Inferiority-self-acceptance	−.22 [.04, .04]	**.46** [.04, .04]	−.44 [.05, .05]	−.19 [.04, .05]
Practical-mindedness	.03 [.04, .04]	−.51 [.05, .05]	−.03 [.05, .06]	.26 [.05, .05]
Optimism-pessimism	.28 [.05, .05]	−.48 [.05, .05]	.16 [.05, .05]	−.01 [.05, .05]
Meticulousness	.01 [.05, .05]	−.54 [.05, .06]	−.18 [.04, .04]	.05 [.05, .05]
Face	.06 [.06, .06]	**.28** [.06, .06]	−.31 [.05, .05]	.24 [.06, .06]
Internal-external-control	.02 [.05, .05]	−.22 [.05, .05]	.37 [.04, .04]	−.07 [.05, .06]
Family-orientation	−.02 [.04, .04]	−.38 [.05, .05]	.28 [.06, .06]	.38 [.05, .05]
Defensiveness	.14 [.04, .04]	.26 [.04, .04]	−.64 [.05, .05]	−.17 [.06, .06]
Graciousness-meanness	−.05 [.04, .04]	−.24 [.05, .05]	**.62** [.05, .05]	.23 [.06, .06]
Interpersonal-tolerance	.20 [.05, .05]	−.08 [.05, .05]	**.54** [.04, .04]	.16 [.05, .05]
Self-social-orientation	.21 [.06, .06]	.13 [.06, .06]	−.59 [.05, .05]	−.02 [.06, .07]
Veraciousness-slickness	−.12 [.04, .04]	−.25 [.05, .05]	**.48** [.06, .07]	.38 [.05, .06]
Traditionalism-modernity	−.16 [.05, .05]	−.26 [.06, .06]	−.62 [.05, .05]	**.08** [.05, .05]
Relationship-orientation	.04 [.05, .06]	.07 [.05, .05]	−.09 [.05, .05]	**.72** [.03, .03]
Social-sensitivity	.29 [.05, .06]	.06 [.04, .04]	−.14 [.04, .04]	**.60** [.04, .05]
Discipline	.06 [.04, .04]	−.18 [.05, .05]	−.73 [.04, .04]	**.30** [.05, .05]
Harmony	−.03 [.04, .04]	−.25 [.04, .04]	.17 [.05, .06]	**.65** [.04, .04]
Thrift-extravagance	−.14 [.06, .06]	−.07 [.07, .07]	−.19 [.06, .06]	**.36** [.06, .06]
	Rotated factor correlations			
	Socpot	Depend	Accom	Interper
Socplot	1			
Depend	−.10 [.04, .04]	1		
Accom	−.06 [.04, .04]	−.31 [.03, .03]	1	
Interper	.21 [.04, .04]	−.30 [.04, .04]	.14 [.03, .03]	1

Note. The table presents point estimates and two types of *SE* estimates (sandwich and bootstrap, in the parentheses). Socpot = social potency; depend = dependability; accom = accommodation; interper = interpersonal relatedness.

A Simulation Study

The Design of the Simulation Study

The goal of the simulation study is to assess the influence of model error and data distributions on *SE* estimates and CIs for factor loadings and factor correlations. The authors consider four levels of model error (RMSEA = 0.00, 0.05, 0.08, and 0.10), three distributions (a normal distribution, an elliptical distribution, and a skewed distribution), and five levels of sample size ($N = 100$, $N = 200$, $N = 537$, $N = 800$, and $N = 2{,}000$). One thousand random samples are generated in each of the 60 conditions. Note that the four levels of the RMSEA correspond to perfect fit, close fit, acceptable fit, and unacceptable fit (Browne & Cudeck, 1993). The population

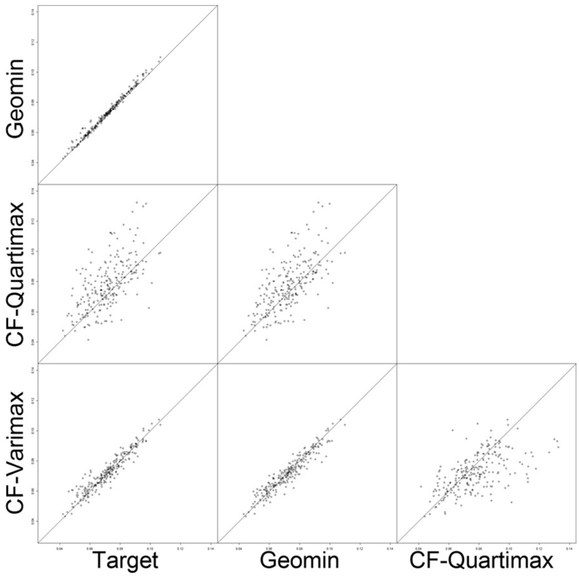

Figure 1. Comparisons of *SE* estimates under four rotation methods with ordinal data (Luo, 2005).
Note. CF = Crawford-Ferguson.

parameters of the simulation study are chosen to be the parameter estimates of an empirical illustration reported earlier (Luo et al., 2008). The sample size of the empirical study was $N = 537$; the authors include four other sample sizes so they can generalize the results to a wider range of conditions.

Let $\widehat{\mathbf{P}}$ be the model implied correlation matrix computed using $\widehat{\boldsymbol{\Lambda}}$ and $\widehat{\boldsymbol{\Phi}}$ of Table 1. We can construct population correlation matrices $\mathbf{P}$ according to a method described by Yuan and Hayashi (2003):

$$\mathbf{P} = \widehat{\mathbf{P}} + \tau\left(\mathbf{R} - \widehat{\mathbf{P}}\right). \tag{3}$$

Here, $\mathbf{R}$ is the sample correlation matrix; τ is a positive number that controls the amount of model error. When $\tau = 0$, the population correlation matrix $\mathbf{P} = \widehat{\mathbf{P}}$; the RMSEA is 0 and no model approximation error is present in the population. Adjusting τ produces three levels of model error: RMSEA values of 0.05, 0.08, and 0.10. Although the population correlation matrices are different at these four levels of model error,[6] factor analysis of these correlation matrices produces the same parameter values.

The three distributions are a normal distribution, an elliptical distribution, and a skewed distribution. In the normal distribution condition, manifest variables are generated from a multivariate normal distribution with a null mean vector and a covariance matrix $\mathbf{P}$. In the elliptical distribution condition, manifest variables are generated from a mixture of two multivariate normal distributions (Ichikawa & Konishi, 1995). Both the normal distribution and the elliptical distribution are symmetric, but the elliptical distribution has heavier tails than the normal distribution. In the skewed distribution, manifest variables are generated using a method described in Yuan and Bentler (1997). The skewed distribution differs from the normal distribution in two ways: It is no longer a symmetric distribution; the marginal kurtosis of different components of

Table 2. Factor Analysis of Ordinal Data (Luo, 2005).

		Factor loadings				
		E	A	C	N	O
E-items	Talkative	**.785** (.057)	.087 (.088)	−.028 (.052)	.197 (.080)	.087 (.065)
	Reserved (R)	−.570 (.067)	−.037 (.068)	.067 (.075)	.146 (.080)	.028 (.061)
	Full Energy	**.499** (.083)	.361 (.118)	.072 (.072)	−.169 (.081)	.094 (.080)
	Enthusiastic	**.579** (.088)	.422 (.097)	−.069 (.065)	−.023 (.053)	.245 (.068)
	Quiet (R)	−.853 (.047)	.006 (.067)	−.036 (.063)	−.041 (.064)	.012 (.052)
	Assertive	**.539** (.073)	−.107 (.093)	.148 (.078)	−.091 (.072)	.303 (.080)
	Shy (R)	−.711 (.056)	.118 (.067)	−.096 (.077)	.126 (.074)	.099 (.066)
	Outgoing	**.775** (.058)	.261 (.101)	.022 (.050)	.019 (.050)	−.039 (.055)
A-items	Find Fault (R)	.007 (.079)	−.395 (.091)	−.018 (.082)	.296 (.099)	.009 (.072)
	Helpful	−.054 (.085)	**.536** (.087)	.088 (.093)	−.031 (.082)	.086 (.080)
	Quarrels (R)	.205 (.109)	−.589 (.099)	−.056 (.088)	.337 (.116)	.038 (.058)
	Forgiving	.011 (.075)	**.629** (.063)	−.081 (.079)	−.054 (.076)	.055 (.081)
	Trusting	.118 (.098)	**.667** (.094)	.025 (.098)	−.017 (.066)	−.070 (.083)
	Cold (R)	−.083 (.112)	−.742 (.072)	.067 (.086)	.213 (.112)	.162 (.072)
	Considerate	−.210 (.112)	**.741** (.061)	.019 (.066)	.040 (.055)	.271 (.075)
	Rude (R)	.168 (.104)	−.531 (.093)	−.062 (.088)	.388 (.107)	−.027 (.054)
	Cooperative	.167 (.112)	**.663** (.081)	.178 (.095)	.087 (.072)	.000 (.053)
C-items	Thorough	−.009 (.057)	.069 (.099)	**.762** (.082)	−.033 (.058)	.091 (.067)
	Careless (R)	.047 (.072)	.014 (.062)	−.496 (.099)	.250 (.094)	.220 (.083)
	Reliable	.046 (.071)	.278 (.132)	**.574** (.099)	.091 (.091)	.043 (.064)
	Disorganized (R)	.079 (.057)	.026 (.054)	−.729 (.079)	.014 (.085)	.193 (.101)
	Lazy (R)	−.025 (.083)	−.114 (.070)	−.538 (.079)	.121 (.085)	.037 (.072)
	Persevere	−.097 (.074)	−.014 (.071)	**.590** (.083)	−.031 (.065)	.234 (.085)
	Efficient	.025 (.070)	.148 (.126)	**.634** (.093)	−.026 (.062)	.145 (.084)
	Plans	.109 (.081)	.129 (.132)	**.664** (.089)	.093 (.071)	.003 (.054)
	Distracted (R)	−.024 (.071)	.153 (.074)	−.382 (.080)	.491 (.065)	.031 (.054)
N-items	Blue	−.123 (.076)	−.190 (.092)	−.213 (.087)	**.563** (.071)	.126 (.071)
	Relaxed (R)	−.027 (.068)	.037 (.069)	.029 (.094)	−.718 (.071)	.023 (.075)
	Tense	−.050 (.059)	−.043 (.052)	.083 (.072)	**.777** (.054)	.159 (.078)
	Worries	−.033 (.047)	.096 (.066)	−.049 (.065)	**.773** (.064)	−.116 (.082)
	Emotionally Stable (R)	−.019 (.051)	.141 (.084)	−.047 (.078)	−.649 (.077)	.145 (.079)
	Moody	.086 (.079)	−.074 (.086)	.017 (.071)	**.751** (.059)	−.010 (.060)
	Calm (R)	.061 (.083)	.059 (.091)	.123 (.106)	−.544 (.079)	.075 (.073)
	Nervous	−.242 (.074)	.209 (.074)	.040 (.057)	**.666** (.062)	−.118 (.065)
O-items	Ideas	.157 (.077)	−.048 (.058)	.090 (.102)	−.041 (.072)	**.700** (.057)
	Curious	.191 (.111)	.165 (.089)	.010 (.086)	−.030 (.076)	**.468** (.076)
	Ingenious	.054 (.075)	−.061 (.076)	.183 (.096)	.030 (.065)	**.601** (.061)
	Imaginative	.122 (.089)	.052 (.069)	−.075 (.093)	.049 (.061)	**.712** (.051)
	Inventive	−.007 (.042)	−.066 (.053)	.026 (.078)	−.159 (.076)	**.793** (.044)
	Artistic	−.145 (.130)	.111 (.093)	−.018 (.079)	.027 (.066)	**.666** (.075)
	Routine (R)	.057 (.086)	.025 (.085)	.196 (.091)	.141 (.086)	−.233 (.071)
	Reflect	.173 (.089)	.064 (.068)	−.075 (.096)	.060 (.072)	**.686** (.058)
	Nonartistic (R)	.052 (.100)	.059 (.083)	−.083 (.082)	.093 (.087)	−.417 (.091)
	Sophisticated	−.264 (.115)	.057 (.090)	.001 (.072)	−.034 (.065)	**.635** (.069)

		Factor correlations				
		E	A	C	N	O
	E	1				
	A	.138 (.059)	1			
	C	.128 (.092)	.288 (.084)	1		
	N	−.249 (.071)	−.193 (.088)	−.302 (.070)	1	
	O	.196 (.076)	.146(.085)	.090 (.089)	−.121 (.080)	1

Note. OLS was used to extract five factors from polychoric correlations of ordinal variables; the factor rotation method is oblique geomin. The table presents point estimates and *SE*s (in parentheses). (R) = reverse-coded items. OLS = ordinary least squares.

Table 3. Average Empirical Coverage Rates of CIs, Target Rotation.

The mean empirical coverage rate across all parameters

	RMSEA	.00			.05			.08			.10		
Dist	*N*	Info	Sand	Boot	Info	Sand	Boot	Info	Sand	Boot	Info	Sand	Boot
N	100	92.7	95.3	96.2	90.8	95.7	96.6	86.3	94.2	95.8	80.7	92.1	94.8
	200	94.2	95.1	95.6	92.8	95.2	96.0	89.2	95.2	96.5	82.8	93.7	95.4
	537	94.8	95.0	95.2	93.3	95.1	95.1	90.7	95.0	95.5	85.5	95.0	96.3
	800	94.8	95.1	95.1	93.5	95.0	95.1	90.6	95.1	95.3	86.7	95.0	96.3
	2,000	95.0	95.1	95.0	93.6	94.9	95.1	91.0	95.1	95.2	87.1	95.1	95.5
E	100	86.8	94.7	96.2	84.0	94.7	95.9	79.5	93.2	95.2	74.4	91.8	93.9
	200	89.6	95.0	95.8	87.6	95.2	96.4	83.0	94.5	96.2	76.6	92.2	95.2
	537	90.4	95.0	95.1	88.6	95.1	95.3	85.0	95.1	95.8	79.5	94.5	96.1
	800	90.7	95.0	95.2	88.9	94.9	95.2	85.7	95.2	95.6	80.3	94.8	96.4
	2,000	91.0	95.1	95.1	88.9	95.0	95.1	85.9	95.2	95.0	81.8	95.2	95.7
S	100	86.5	94.1	96.5	81.2	93.4	95.3	76.0	91.6	93.7	71.7	89.8	92.2
	200	89.2	94.8	96.2	85.6	94.8	97.0	78.3	93.3	95.7	72.0	90.9	93.8
	537	90.2	94.7	95.2	89.6	94.9	96.4	83.7	94.3	96.9	74.2	91.9	95.1
	800	90.3	94.6	95.0	90.0	94.9	95.9	84.8	95.0	97.0	77.5	93.2	96.2
	2,000	90.4	94.8	94.9	90.1	94.8	95.2	86.8	95.2	96.3	81.0	95.0	97.2

Note. CI = confidence interval; RMSEA = the root mean square error of approximation; Emp = empirical *SEs*; Info = *SEs* with the information matrix; Sand = sandwich *SEs*; Boot = bootstrap *SEs; N* = normal distribution; E = elliptical distribution; S = skewed distribution.

y are different. Note that the population correlation matrix is the same in all three distribution conditions.

Results of the Simulation Study

In each simulated sample, the authors extract four factors using ML and conduct oblique rotation using both CF-varimax and target rotation. They compute *SE*s for factor loadings and factor correlations using three methods: the information method, the sandwich method, and the bootstrap method. They construct CIs using point estimates and *SE* estimates.

Let $[\widehat{\theta}_L, \widehat{\theta}_U]$ be the CI for θ. Its empirical coverage rate over 1,000 simulation samples is the proportion of samples whose CI includes θ. Table 3 reports the mean empirical coverage rates (averaged across all factor loadings and factor correlations) of three types of CIs (sandwich CIs, information CIs, and bootstrap CIs) for oblique target rotation.[7]

Four observations can be made on the mean empirical coverage rates of the three types of CIs. First, all three types of CIs have satisfactory empirical coverage rates under the ideal condition of normally distributed variables, no model error (RMSEA $=0.00$), and moderately large samples ($N \geq 200$). Second, as the amount of model error increases, the empirical coverage rates of information CIs decrease. Such decreases are more pronounced when RMSEA $=0.08$ and RMSEA $=0.10$. The empirical coverage rates of sandwich CIs and bootstrap CIs are closer to 95% regardless of the amount of model error. Third, the empirical coverage rates of information CIs for nonnormal data are lower than those for normal data. The empirical coverage rates of sandwich CIs and bootstrap CIs are close to 95% regardless of data distributions. Fourth, increasing sample size makes empirical coverage rates of sandwich CIs and bootstrap CIs close to 95% in model error and nonnormal data conditions, but increasing sample size does not improve the empirical coverage rates of information CIs in such conditions.

Table 4. Average Empirical Coverage Rates of CIs, CF-Varimax.

The mean empirical coverage rate across all parameters

	RMSEA	.00			.05			.08			.10		
Dist	*N*	Info	Sand	Boot	Info	Sand	Boot	Info	Sand	Boot	Info	Sand	Boot
N	100	92.2	94.5	97.6	87.9	93.5	96.8	82.9	91.9	96.2	78.1	89.8	94.9
	200	93.7	94.8	97.3	92.6	95.5	96.9	85.6	93.4	97.5	80.8	91.9	96.3
	537	94.5	95.4	96.1	93.1	95.2	96.3	89.8	95.2	96.7	83.5	94.2	96.7
	800	94.6	94.9	95.7	93.5	95.5	96.2	90.3	95.6	96.4	84.5	95.3	97.3
	2,000	94.8	95.1	95.1	93.2	95.0	95.5	90.3	95.1	95.3	85.1	95.0	96.1
E	100	85.2	93.8	96.5	82.3	93.5	96.4	76.2	91.5	94.8	73.4	90.1	93.6
	200	89.9	95.3	96.8	86.4	94.7	97.3	79.9	93.3	96.6	73.9	91.4	94.9
	537	90.5	95.1	96.4	88.5	95.0	96.5	84.0	95.2	96.9	77.0	93.2	96.6
	800	91.4	95.4	95.8	89.1	95.7	95.9	84.3	95.4	96.8	77.3	94.7	96.9
	2,000	91.2	95.1	95.1	88.6	95.1	95.3	85.0	95.1	95.4	79.7	95.6	96.5
S	100	85.2	93.3	93.5	79.7	92.3	95.1	75.0	91.1	92.7	70.7	88.4	92.2
	200	88.9	94.1	97.4	84.1	93.4	97.2	76.2	90.8	94.6	69.9	89.0	93.0
	537	90.4	94.8	96.0	88.7	94.9	97.4	82.2	94.5	97.7	73.8	90.3	96.0
	800	90.5	95.2	95.6	88.9	94.8	96.8	83.8	94.9	97.7	73.2	91.6	96.8
	2,000	90.2	94.9	95.0	89.7	95.2	95.5	86.1	95.0	96.4	78.9	94.4	97.5

Note. CI = confidence interval; CF = Crawford-Ferguson; RMSEA = the root mean square error of approximation; Emp = empirical *SE*s; Info = *SE*s with the information matrix; Sand = sandwich *SE*s; Boot = bootstrap *SEs; N* = normal distribution; E = elliptical distribution; S = skewed distribution.

Table 4 reports the mean empirical coverage rates (averaged across all factor loadings and factor correlations) of three types of CIs (sandwich CIs, information CIs, and bootstrap CIs) for oblique CF-varimax rotation. Although the four observations made on target rotation apply to CF-varimax rotation, the sandwich CIs and the bootstrap CIs perform less satisfactorily for CF-varimax rotation than for target rotation. The empirical coverage rates of sandwich CIs are lower than 95% for small samples and move closer to 95% at larger samples; the phenomenon is particularly noticeable when the amounts of model error are larger (RMSEA $=0.08$ and RMSEA $=0.10$). The empirical coverage rates of bootstrap CIs are higher than 95% in most conditions.

The relative advantage of target rotation over CF-varimax rotation is expected. CF-varimax rotation is an automatic rotation method, but target rotation requires the factor analyst to provide a target matrix that reflects substantive knowledge about the factor loading pattern. Regardless of factor rotation methods, data distributions, and levels of model error, the sandwich method provides useful *SE* estimates and CIs at a moderately large sample size.

Concluding Comments

A parsimonious EFA model is unlikely to perfectly represent complicated real-world phenomena, and model error is always present in EFA (MacCallum, 2003). Let us consider a hypothetical scenario in which 10 manifest variables are affected by two major factors and 30 minor factors. Only the two major factors have large factor loadings and the 30 minor factors have only small loadings. A useful factor analysis model does not fit data perfectly, but it captures the influence of major common factors with the presence of minor factors that are like background noise. The information *SE*s and CIs may be invalid with model error, but sandwich *SE*s and CIs are still valid. Factor analysts can interpret rotated factor loadings and factor correlations by examining their sandwich CIs.

A common reason for nonnormal distributions is the use of ordinal variables. To accommodate ordinal variables, the polychoric correlation matrix is factor analyzed. Because polychoric correlation matrices are often not positive definite, ML estimation is infeasible. The authors consider OLS estimation for its computational robustness. Although estimating factor loadings and factor correlations involves only polychoric correlations, estimating *SE*s and CIs involves the asymptotic covariance matrix of polychoric correlations. Estimating such a large matrix is a nontrivial task. For example, there are 44 ordinal variables in the second empirical study, and the corresponding asymptotic covariance matrix has nearly half a million nonduplicated elements. The authors implemented an algorithm that uses an estimating equation approach (Yuan & Schuster, 2013) to estimate the asymptotic covariances of polychoric correlations.

The sandwich *SE* estimator is more versatile than the bootstrap method (Ichikawa & Konishi, 1995) and the infinitesimal jackknife method (Zhang et al., 2012). The bootstrap method is inappropriate for geomin rotation, which tends to produce multiple local solutions (Browne, 2001; Hattori, Zhang, & Preacher, 2017). The infinitesimal jackknife method is equivalent to the sandwich *SE* estimator when manifest variables are continuous, but it is inappropriate for ordinal variables. The sandwich *SE* estimator can be adapted in both situations.

The R package EFAutilities (Zhang et al., 2018) implements the sandwich *SE* estimator and the corresponding CIs. It computes *SE*s and CIs for EFA parameters with normal and nonnormal data, two types of estimation method (ML and OLS), and oblique rotation and orthogonal rotation with four rotation criteria (CF-varimax, CF-quartimax, geomin, or target), with any level of model approximation error.

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Notes

1. It is commonly referred to as the normal theory based *SE*. The authors avoid this name because it ignores the assumption of a perfectly fitting model in the population.
2. Most EFA models are estimated with sample correlation matrices. If a factor analyst is interested in estimating EFA models with a sample covariance matrix, the sandwich method can be easily adapted. The adaptation involves replacing $\mathbf{\Gamma}$ by the asymptotic covariance matrix of unique elements of a sample covariance matrix. All other components remain the same.
3. The address for the online file is https://www3.nd.edu/~gzhang3/Papers/SandwichEFA/SandwichEFA.html
4. The authors include comparisons between information *SE*s and sandwich *SE*s in the online supporting file. For nearly all parameters, sandwich *SE* estimates are larger than the corresponding information *SE* estimates.
5. The authors thank Shanhong Luo for making the data set available to them.
6. The four population correlation matrices are included in the online supporting file.
7. The results for CIs and *SE* estimates for single parameters are included in the online file.

Supplemental Material

Supplemental material is available for this article online.

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