

Homework 1  
Answers

1. The name of R for release 5.6.1 is “\_\_\_\_\_”.
2. (From Gelman) Conditional Probability: Approximately 1/125 of all births are fraternal twins and 1/300 of births are identical twins. Elvis Presley had a twin brother (who died at birth). What is the probability that Elvis was an identical twin? (approximate the probability of a boy or girl = 1/2).

Given:

Elvis's twin is a boy

$$P(\text{Girl}) = P(\text{Boy}) = 1/2$$

$$P(\text{identical}) = (1/300)$$

$$P(\text{fraternal}) = (1/125)$$

Can Find:

Since we know that for identical twins, the siblings are either both are boys or both are girls,

$$P(\text{brother \& identical}) = P(\text{brother}|\text{identical})P(\text{identical}) = (1/2)(1/300)$$

Since we know that for fraternal twins, they can be both boys, boy girls, boy(first born)-girl(second born), or girl(first born)-boy(second born), then there is only 1 out of 4 ways to have two boys,

$$P(\text{brother \& fraternal}) = P(\text{brother}|\text{fraternal})P(\text{fraternal}) = (1/4)(1/125)$$

$$\begin{aligned}
P(\text{twin is a brother}) &= P(\text{brother \& identical}) + P(\text{both boys \& fraternal}) \\
&= P(\text{brother}|\text{identical})P(\text{identical}) + P(\text{brother}|\text{fraternal})P(\text{fraternal}) \\
&= (1/2)(1/300) + (1/4)(1/125) \\
&= .0036\bar{6}
\end{aligned}$$

Answer:

$$\begin{aligned}
P(\text{identical twin}|\text{brother}) &= \frac{P(\text{brother \& identical})}{P(\text{twin brother})} \\
&= \frac{(1/2)(1/300)}{(1/2)(1/300) + (1/4)(1/125)} \\
&= .00166\bar{6} \\
&= .41
\end{aligned}$$

From google search,

On January 8, 1935, Elvis Aron (later spelled Aaron) Presley was born at his parents' two-room house in East Tupelo, Mississippi, about 35 minutes after his identical twin brother, Jesse Garon, who was stillborn. The next day, Jesse was buried in an unmarked grave in nearby Priceville Cemetery. Jan 8, 2019.

3. Mathematical Models of Probabilities: Kruschke page 31, exercise 2.1  
Given:

- 4 sided die with  $x = 1, 2, 3, \text{ or } 4$  (number of dots)
- Model A:  $p(x) = 1/4$
- Model B:  $p(x) = x/10$
- Model C:  $p(x) = 12/(25x)$

Describe in words what kind of bias (or lack of bias) is expressed by each model

$x$	Model A $p(x A) = 1/4$	Model B $p(x) = x/10$	Model C $p(x) = 12/(25x)$
1	.25	.10	.48
2	.25	.20	.24
3	.25	.30	.15
4	.25	.40	.12

**Model A** is “flat”; that is, all probabilities are the same and equal to .25. There is no bias.

**Model B** has a negative skew ; that is, higher numbers are more likely. There is a bias in favor of larger numbers.

**Model C** has a positive skew; that is, smaller numbers are more likely. There is a bias in favor of small numbers.

4. How Data Shifts Probability: Kruschke, page 31-32, exercise 2.2.

Since we have no knowledge of which Model might be true, our priors are flat; that is,  $P(A \text{ true})=P(B \text{ true})=P(C \text{ true}) = 1/3$ .

- (a) Data are  $x_1 = 25, x_2 = 25, x_3 = 25, \text{ and } x_4 = 25$ . This is out of  $n = 100$  independent rolls.

Model A is most consistent with the data. Model B and Model C are both skewed distributions and neither is consistent with the data.

- (b) Data are  $x_1=48, x_2=24, x_3 = 15, \text{ and } x_4 = 12$ . This is out of  $n = 100$  independent rolls.

These are the values that Model C would predict (on average) and is most consistent with the data. Model B is definitely the worst because the skew is opposite that of Model C. Model A is in between Model C (best) and B(worst).

To formally use Bayes, need the data model, multinomial, and it's conjugate prior would be Dirletht distribution.