EdPsych/Psych/Stat 587
C.J. Anderson

SAS: Homework 5
Answer Key

1. *Report $-$LnLike, number of parameters, AIC, HQIC, BIC, CAIC for all of the model fit in computer labs 1 and 2. Report $\hat{\tau}_0^2$, σ^2, R_1^2 and R_2^2 (a table of this information would be very nice).*

   See attached table.

   (a) *Based on the information criteria, among the random intercept models, which is the "best" (or is there a unique one)?*

   There is not a unique best model. The smallest values for the random intercept models are underlined in the table.

   - According to the AIC, model (b) from lab 1/homework1, but this is a fixed effects model.
   - Among random intercept models, model (l) from lab 2/homework 2 have the smallest AIC.
   - According to the HQIC, BIC and CAIC, model (k) from computer lab 2/homework 2 is the "best."

   (b) *Based on the information criteria, among* ALL *models (random intercept and random slopes models), which is the "best" (or is there a unique one)?*

   In this case, there is a unique "best" model. Model (p) has the smallest value of for the AIC (48291.4), HQIC (48313.2), BIC (48345.1) and CAIC (48363.1).

   (c) *What is the value of the harmonic mean used to compute R_2^2?*

   Harmonic mean:
   $$\bar{x}_+ = 146/3.627 = 40.2521$$

   (d) *Which is the "best" model based on R^2 measures? Are the models with the better R^2's the same as the better/best models according to the information criteria?*

   In general, the smaller information criteria go with the larger R_1^2 and R_2^2; however, they don't "select" the same "best" model. The better models are random intercept model (l) from Homework 2 and beyond. These all have $R_1^2 = .45$ and $R_2^2 = .80$, except for model (s) where $R_2^2 = .79$. The R_1^2 and R_2^2 don't distinguish among these models (unless we went out to more decimal places, but this doesn't seem to be warranted).

(e) *Interpret the values of R_1^2 and R_2^2 from model (p).*

- $R_1^2 = .45$: The proportional reduction in (squared) prediction errors of Y_{ij}'s using group centered math, gender, grade, hours watching TV, etc. versus the model with no predictor variables is 45%.
- $R2_2^2 = .80$: The proportional reduction in (squared) prediction errors of the group means $\bar{Y}_{+j}$ using predictor variables relative to the null model (i.e., no predictor variables) is 80%.

Model	$-2\times$ LnLike	#	AIC	HQIC	BIC	CAIC	$\hat{\tau}$s	$\hat{\sigma}^2$	R_1^2	R_2^2
From Lab/homework 1										
Fixed Effects Models										
(a)	50964.1	147	51258.1	51605.8	52267.6	52414.6	n.a.	76.9537	n.a.	n.a.
(b)	48061.8	148	$\underline{48357.8}$	48707.8	49374.2	49522.2	—	51.1241	—	—.
Random Effects (Intercept) Models										
(c)	51489.0	3	51495.0	51498.7	51504.0	51507.0	$\tau_0^2 = 20.9437$	78.5815	*99.52**	*22.90†*
(d)	51489.0	3	51495.0	51498.7	51504.0	51507.0	$\tau_0^2 = 20.9437$	78.5815		
(e)	48482.4	4	48490.4	48495.2	48502.3	48506.3	$\tau_0^2 = 6.0149$	52.2296	.41	.68
(f)	48482.4	4	48490.4	48495.2	48502.3	48506.3	$\tau_0^2 = 6.0149$	52.2296	.41	.68
(g)	48646.8	4	48654.8	48659.6	48666.7	48670.7	$\tau_0^2 = 21.6691$	52.2039	.26	.00
(h)	51267.2	4	51275.2	51280.1	51287.2	51291.2	$\tau_0^2 = 3.0484$	78.6205	.18	.78
(i)	48426.4	5	48436.4	48442.5	48451.3	48456.3	$\tau_0^2 = 3.7105$	52.2304	.43	.78
(j)	48363.7	7	48377.7	48386.2	48398.6	48405.6	$\tau_0^2 = 3.7542$	51.7541	.44	.78
(k)	48359.0	10	48379.0	48391.1	48403.3	48418.8	$\tau_0^2 = 3.6278$	51.7470	.44	.79
(l)	48363.5	8	48379.5	48389.2	48403.4	48411.4	$\tau_0^2 = 3.7509$	51.7533	.44	.78
(m)	student's choice									
From Lab/homework 2										
Random Intercept										
(j)	48363.7	7	48377.7	48386.2	48398.6	48405.6	$\tau_0^2 = 3.7542$	51.7541	.44	.78
(n)	48347.2	9	48365.2	$\underline{48376.1}$	$\underline{48392.0}$	$\underline{48401.0}$	$\tau_0^2 = 3.6654$	51.6515	.44	.78
(o)	48338.6	12	48362.6	48377.2	48398.4	48410.4	$\tau_0^2 = 3.3762$	51.6546	.45	.80
Random Intercept & Slopes										
(p)	48306.8	14	48334.8	48351.8	48376.6	48390.6	$\tau_0^2 = 3.44$	51.4261	.45	.79
							$\tau_{10} = 0.06$			
							$\tau_1^2 = 0.01$			
(q)	48334.1	14	48362.1	48379.0	48403.8	48417.8	$\tau_0^2 = 3.69$	51.4261	.45	.80
							$\tau_{10} = 0.31$			
							$\tau_1^2 = 0.16$			
(r)	48337.7	14	48365.7	48382.7	48407.5	48421.5	$\tau_0^2 = 5.06$	51.6365	.45	.80
							$\tau_{10} = 0.19$			
							$\tau_1^2 = 0.02$			
(s)	48255.4	18	$\boxed{48291.4}$	$\boxed{48313.2}$	$\boxed{48345.1}$	$\boxed{48363.1}$	$\tau_0^2 = 3.40$	50.9033	.45	.80
							$\tau_{10} = 0.04$			
							$\tau_1^2 = 0.002$			
(t)	48255.1	19	48293.1	48316.1	48349.8	48368.8	$\tau_0^2 = 3.40$	50.8984	.45	.80
							$\tau_{10} = .0.044$			
							$\tau_1^2 = 0.023$			
(u)	48247.3	21	48289.3	48314.7	48351.9	48372.9	$\tau_0^2 = 4.1692$	50.8984	0.45	0.80
							$\tau_{10} = 0.096$			
							$\tau_1^2 = 0.023$			
							$\tau_{40} = -0.1607$			
							$\tau_{14} = -0.0167$			
							$\tau_4^2 = 0.1289$			
(v)	Student's choice									

Model	-2LnLike	#	AIC	HQIC	BIC	CAIC	$\hat{\tau}_0^2$	$\hat{\sigma}^2$	R_1^2	R_2^2
Model Refinements										
Model (s) without random slope										
(s)	48261.3	16	48293.4	48312.7	48341.1	48357.1	3.3957	51.0818		
Model (s) Dropped cross-level interaction with type of community										
(s*)	48260.0	15	48290.0	48308.2	48334.8	48349.8	not reported here		.45	.80
Model (s*) Re-coded community type into 2 levels										
(s**)	48262.2	13	48288.2	48304.0	48327.0	48340.0			.45	.79
Model (s**) hours TV as class variable										
(w)	48242.8	16	48274.8	48294.2	48322.5	48338.5			.45	.80
Model ( ) with hours TV recoded into 2 levels										
(x)	48248.8	13	48274.8	48290.6	48313.6	48326			.45	.80
Model ( ) with hours playing computer games as class										
(y)	48237.0	16	48269.0	48288.4	48316.8	48332.8			.45	.80
Model ( ) with hours playing computer games recoded into 2 levels										
(z)	**48240.0**	**13**	**48266.0**	**48281.7**	48304.7	48317.7			.45	.80
Model (z) without a random slope										
(z-)	48246.2	11	48268.2	**48281.5**	**48301.0**	**48312.0**	3.3517	50.9817		

$* = \hat{\tau}_0^2 + \hat{\sigma}^2 = 99.5252$ from null model. $\dagger = \hat{\tau}_0^2 + \hat{\sigma}^2/\bar{x}_+ = 22.8959$ from null model.

2. *Consider model (s) from computer lab2/homework 4. Do you need a random slope? (Be sure to report the statistical test that you use for this question).*

Test for random slope: $H_o : \tau_1^2 = \tau_{10} = 0$ versus H_a : no H_o.

Test statistic ($\mathcal{M}_o$ is model (s) without random slope and $\mathcal{M}_a$ is model (s) :

$$\begin{aligned}\lambda &= -2(\text{LnLike}\mathcal{M}_o) - \text{LnLike}\mathcal{M}_a)) \\ &= 48261.3 - 48255.4 \\ &= 5.9\end{aligned}$$

mixture of p-value from χ_1^2 and χ_2^2 equals .01; reject H_o, the data support the conclusion that we need a random slope for group centered math.

3. *Compare and contrast the standard errors of parameters and results of significance tests for fixed effects when you use the model based versus the robust estimators of the standard errors. Which do you think is the best to use for testing fixed effects and why?*

   Note: For this problem, just look at model (s) and in this case use the ddfm=betwithin method for getting df because they at not computed using Satterthwaite.

Effect	Estimate	DF	Model Based Standard Error	Pr > $\|t\|$	Robust Standard Error	Pr > $\|t\|$
Intercept	15.8565	141	5.8623	.0077	6.0107	.0093
grpCmath	3.3033	6942	0.3548	< .0001	0.3490	< .0001
gender	1.2766	145	0.1724	< .0001	0.1835	< .0001
gender	0	.	.	.	. .	.
grade	-0.8738	135	0.1941	< .0001	0.2083	< .0001
grade	0	.	.	.	. .	.
hours–TV	-0.09139	6942	0.07410	.2175	0.08203	.2652
hours–computergames	-0.2557	6942	0.07819	.0011	0.07858	.0011
grpMmath	0.8913	141	0.03892	< .0001	0.04018	< .0001
type–community (isolated)	4.3284	141	2.0471	.0362	0.2662	< .0001
type–community (rural)	1.0834	141	0.5079	.0347	0.4553	.0187
type–community (suburb)	0.07886	141	0.4032	.8452	0.4235	.8525
type–community (urban)	0	.	.	.	. .	.
grpCmath*grpMmath	-0.01827	6942	0.00235	< .0001	0.002316	< .0001
grpCmath*type-commun	0.09869	6942	0.1053	.3487	0.01402	< .0001
grpCmath*type-commun	0.05578	6942	0.03146	.0763	0.02845	.0499
grpCmath*type-commun	0.03444	6942	0.02334	.1401	0.02343	.1416
grpCmath*type-commun	0	.	.	.	.	.

- The robust standard errors for fixed effects tend to be of similar values for the model based ones.
- The notable exceptions: The model based standard errors for isolated communities (2.0471) is considerably different from the empirical one (0.2662) and that for those for the cross-level interactions between group centered math and rural communities are considerably different (empirical = 0.01 and model based = 0.10).
- The results of the significance tests yield same conclusions same, except for rural communities cross-level interaction for rural and grpCmath (model $p = .35$ and empirical $p =< .0001$).
- The standard errors for the covariance parameters are not effected.

4. *Report what contrasts you tested, the results of them, and any action you took based on the results for*

I did the following using both empirical and model based standard errors (in the SAS code); however, I report the model based ones with Satterthwaite degrees of freedom below.

I fit models sequentially according to order of the models reported in the big table.

(a) *Type of community*
See homework answer key 2.. . . and you were guided through this by the computer lab instructions.

Label	Num DF	Den DF	F Value	Pr > F
Urban & Suburban	1	130	0.00	0.9452
Isolated with sub/urban	1	108	3.94	0.0497
Rural with sub/urban	1	144	3.96	0.0486
Isolated vs Rural	1	109	2.20	0.1411

So I re-coded type of community into three levels: isolated, rural, and sub/urban. This model is model (s) in my big table.

(b) *Hours watching TV or videos* After examining the parameter estimates, I decided on to try
SAS INPUT:

```
contrast 'None & Some '
   hours_computer_games 1 -1 0 0 0 ,
   hours_computer_games 1 0 -1 0 0 ,
   hours_computer_games 1 0 0 -1 0 ;
contrast 'Some vs A lot'
   hours_computer_games -.25 -.25 -.25 -.25 1;
```

SAS output:

Contrasts

Label	Num DF	Den DF	F Value	Pr > F
All < 4 hours are same	3	7021	2.03	0.1071
Some vs 4+ hours	1	7039	12.73	0.0004

So, I re-coded hours playing computer games as

$$\text{TV} = \begin{cases} 0 & \text{if } \leq 4 \text{ hrs (some)} \\ 1 & \text{if } > 4 \text{ hrs (a lot)} \end{cases}$$

The fit of model with re-coded TV variables are reported in the big summary table as model (t).

(c) *Hours playing computer games.* I did similar things for this one:

Contrasts

Label	Num DF	Den DF	F Value	Pr > F
All < 4 hours are same	3	7033	0.98	0.4013
Some vs 4+ hours	1	6997	14.95	0.000

Since the big distinction is between a lot versus some, I re-coded hours playing computer games as

$$\text{Computer Games} = \begin{cases} 0 & \text{if } \leq 4 \text{ hrs (some)} \\ 1 & \text{if } > 4 \text{ hrs (a lot)} \end{cases}$$

The results in terms of goodness-of-fit are reported in big table summary table, model (z).

5. *Starting with model (s) from computer lab2/homework 2, refine this model to obtain a "best" model (i.e. simplify by dropping effects, re-coding a discrete variable, etc.). Summarize the steps that you and why you took them. This includes how you used the information from your contrasts, information criteria, tests of parameters, etc.*

Using information from this homework and other analyses, I know the following:

- I need a random intercept.
- I need a random slope for grpCmath.
- I can simplify the model by re-coding type of community, hours watching TV (which is more categorical than numerical), and hours playing computer games (which is more categorical than numerical). I found this from the contrasts.
- Below are refinements done successively to show that re-coding of community is fine, hours watching TV and playing computer games should be treated as categorical variables, and that proposed re-coding is fine. Lastly, I re-tested to see whether I still need a random slope (yes).

Model	-2LnLike	#	λ	*df*	*p*-value	Conclusion
(s)	48255.4	18				
Model (s) w/o cross-level interaction with type of community						
(s*)	48260.0	15	4.6	3	.20	OK to drop cross-level
Model (s*) re-coded community type into 2 levels						
(s**)	48262.2	13	2.2	2	.33	Re-coding of community OK.
Model (n) hours TV as class variable						
(w)	48242.8	16	19.4	3	$< .01$	TV should be class.
Model (o) with hours TV recoded into 2 levels						
(u)	48248.8	13	6.0	3	.11	re-coding of TV OK.
Model (p) with hours playing computer games as class						
(y)	48237.0	16	11.8	3	$< .01$	Computer games should be class.
Model (q) with hours playing computer games recoded into 2 levels						
(z)	48240.0	13	3.0	3	.39	Recoding of games OK
Model (r) without a random slope						
(z-)	48246.2	11	6.2	2/1	.03	Still need random slope

Note: Comparing (z) and (z-), the mixture was between χ^2_1 ($p = .01278$) and χ^2_2 ($p = .04505$).

- Although Model (z) doesn't have the smallest values on all the information criteria, I selected it because
  (a) It has small or almost the smallest values on the IC.
  (b) All of the effects in the model are significant.
  (c) It is parsimonious.
  (d) The interpretation makes sense.

6. *If your final model has a random slope, re-check to make sure that you need it. Report your results.*

   The null hypothesis is $H_o : \tau_1^2 = \tau_{01} = 0$. Model (z) is the "full" model and model (z-) is the "nested" model. The test statistic equals

   $$\lambda = 48246.2 - 48240 = 6.2$$

   Comparing $\lambda = 6.2$ to the χ^2_1, it has a $p = .01$ and compared to χ^2_2 it has a $p = .045$, which gives our test a $p = .5(.01278 + .04505) = .03$. Therefore, reject H_o, the data support that conclusion that we need a random slope[1].

7. *Give a full interpretation of the final model. Also give the HLM, linear mixed model and marginal model formulations using the parameter estimates.*

   Parameter Estimates from model (z):

[1]Note that the z test in the table of covariance parameter estimates (see next item) indicates otherwise).

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm	Subject	Standard Estimate	Z Error	Value	Pr Z
UN(1,1)	idschool	3.3546	0.5425	6.18	<.0001
UN(2,1)	idschool	0.04459	0.02374	1.88	0.0604
UN(2,2)	idschool	0.002365	0.001709	1.38	0.0832
Residual		50.7922	0.8698	58.39	<.0001

Solution for Fixed Effects

Effect		Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		14.4613	5.7171	142	2.53	.0125
grpCmath		3.1863	0.3498	145	9.11	<.0001
gender	boy	1.2952	0.1716	6982	7.55	<.0001
gender	girl	0	.	.	.	.
grade	3	-0.8617	0.1938	6930	-4.45	<.0001
grade	4	0	.	.	.	.
hrsTV	<4 hours	0.8449	0.2280	7040	3.71	.0002
hrsTV	4+ hours	0	.	.	.	.
hrsCGames	<4 hours	1.5226	0.3698	7005	4.12	<.0001
hrsCGames	4+ hours	0	.	.	.	.
community	isol or rural	1.0755	0.4629	143	2.32	.0216
community	sub or urban	0	.	.	.	.
grpMmath		0.8815	0.03796	143	23.22	<.0001
grpCmath*grpMmathath		-0.01740	0.002306	143	-7.54	<.0001

As an HLM:

Hierarchical model :

Level 1 :

$$\begin{aligned}\widehat{\texttt{(science)}}_{ij} &= \hat{\beta}_{0j} + \hat{\beta}_{1j}\texttt{(grpCmath)}_{ij} + 1.30\texttt{(gender)}_{ij} - 0.86\texttt{(grade)}_{ij} \\ &\quad +0.84\texttt{(hours\_TV)}_{ij} + 1.52\texttt{(hours\_computer\_games)}_{ij} + R_{ij}\end{aligned}$$

where $\hat{\sigma}^2 = 50.79$.

Level 2 :

$$\begin{aligned}\hat{\beta}_{0j} &= 14.46 + 0.88\texttt{(grpMmath)}_j + 1.08(\text{community})_j \\ \hat{\beta}_{1j} &= 3.20 - 0.02\texttt{(grpMmath)}_j \\ \hat{\beta}_{2j} &= 1.30 \\ \hat{\beta}_{3j} &= -0.86 \\ \hat{\beta}_{4j} &= 0.84 \\ \hat{\beta}_{5j} &= 1.52\end{aligned}$$

and

$$\hat{\boldsymbol{T}} = \begin{pmatrix} 3.355 & 0.045 \\ 0.045 & 0.002 \end{pmatrix}$$

Linear mixed model :

$$\begin{aligned}\texttt{(science)}_{ij} &= 14.46 + 3.20\texttt{(grpCmath)}_{ij} + 1.30\texttt{(gender)}_{ij} - 0.86\texttt{(grade)}_{ij} \\ &\quad +0.84\texttt{(hours\_TV)}_{ij} + 1.52\texttt{(hours\_computer\_games)}_{ij} \\ &\quad +0.88\texttt{(grpMmath)}_j + 1.08(\text{community})_j + \\ &\quad -0.02\texttt{(grpMmath)}_j\texttt{(grpCmath)}_{ij}\end{aligned}$$

On average, a student's science score is expected to be

1.30 points higher for boys

.86 points higher for 4th graders than 3rd graders

.84 points higher for students who watch TV less than 4 hours per week.

1.52 points higher for students who play computer games less than 4 hours per week.

1.08 points higher when the student's school is either a rural or isolated school.

The effect of math scores on science is $\hat{\beta}_{1j} = 14.46 - 0.02\texttt{(grpMmath)}_j$; that is, for a 1 unit increase of student's math score relative to their peers, the student's science score is expected to be $14.46 - 0.02\texttt{(grpMmath)}_j$. The higher the average math scores in a school, the lower the effect of a student's relative standing.

There are also differences between schools in terms of the intercepts. The overall level of the science scores is influenced by a school's location (isolate or rural school have on average higher science scores than urban or suburban schools) and the mean math scores of students at the school (on average, the overall level of science scores are .88 points higher).