EdPsy/Psych/Stat 587
Spring 2019
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SAS: Homework 4
Answer Key

1. (10 points) For models (n) – (v) that you fit to the TIMSS data in computer lab 2, write out the equation of the model in each of the following ways:

(n) Hierarchical model :

Level 1 :

$$\begin{aligned}\texttt{(science)}_{ij} &= \beta_{0j} + \beta_{1j}\texttt{(grpCmath)}_{ij} + \beta_{2j}\texttt{(gender)}_{ij} + \beta_{3j}\texttt{(grade)}_{ij} \\ &\quad + \beta_{4j}\texttt{(hours\_TV)}_{ij} + \beta_{5j}\texttt{(hours\_computer\_games)}_{ij} + R_{ij}\end{aligned}$$

where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2 :

$$\begin{aligned}\beta_{0j} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_j + U_{0j} \\ \beta_{1j} &= \gamma_{10} \\ \beta_{2j} &= \gamma_{20} \\ \beta_{3j} &= \gamma_{30} \\ \beta_{4j} &= \gamma_{40} \\ \beta_{5j} &= \gamma_{5j}\end{aligned}$$

where $U_{0j} \sim \mathcal{N}(0, \tau_0^2)$ i.i.d and independent of R_{ij}.

Linear mixed model :

$$\begin{aligned}\texttt{(science)}_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\ &\quad + \gamma_{40}\texttt{(hours\_TV)}_{ij} + \gamma_{5j}\texttt{(hours\_computer\_games)}_{ij} + \gamma_{01}\texttt{(grpMmath)}_j \\ &\quad + U_{0j} + R_{ij}\end{aligned}$$

Marginal model : $\texttt{(science)}_{ij} \sim \mathcal{N}(\mu_{ij}, (\tau_0^2 + \sigma^2))$ where

$$\begin{aligned}\mu_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\ &\quad + \gamma_{40}\texttt{(hours\_TV)}_{ij} + \gamma_{5j}\texttt{(hours\_computer\_games)}_{ij} + \gamma_{01}\texttt{(grpMmath)}_j\end{aligned}$$

(o) Since type of community is a discrete and nominal variable, we'll need 3 dummy (or effect) codes for this variable. I'll use dummy codes defined as

$$\texttt{(isolated)}_j = \begin{cases} 1 & \text{if isolated} \\ 0 & \text{otherwise} \end{cases} \quad \texttt{(rural)}_j = \begin{cases} 1 & \text{if rural} \\ 0 & \text{otherwise} \end{cases} \quad \texttt{(suburb)}_j = \begin{cases} 1 & \text{if suburban} \\ 0 & \text{otherwise} \end{cases}$$

Hierarchical model :

Level 1 :

$$\begin{aligned} \texttt{(science)}_{ij} &= \beta_{0j} + \beta_{1j}\texttt{(grpCmath)}_{ij} + \beta_{2j}\texttt{(gender)}_{ij} + \beta_{3j}\texttt{(grade)}_{ij} \\ &\quad + \beta_{4j}\texttt{(hours\_TV)}_{ij} + \beta_{5j}\texttt{(hours\_computer\_games)}_{ij} + R_{ij} \end{aligned}$$

where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2 :

$$\begin{aligned} \beta_{0j} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(isolated)}_j + \gamma_{03}\texttt{(rural)}_j \\ &\quad + \gamma_{04}\texttt{(suburb)}_j + U_{0j} \\ \beta_{1j} &= \gamma_{10} \\ \beta_{2j} &= \gamma_{20} \\ \beta_{3j} &= \gamma_{30} \\ \beta_{4j} &= \gamma_{40} \\ \beta_{5j} &= \gamma_{5j} \end{aligned}$$

where $U_{0j} \sim \mathcal{N}(0, \tau_0^2)$ i.i.d and independent of R_{ij}.

Linear mixed model :

$$\begin{aligned} \texttt{(science)}_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\ &\quad + \gamma_{40}\texttt{(hours\_TV)}_{ij} + \gamma_{5j}\texttt{(hours\_computer\_games)}_{ij} \\ &\quad + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(isolated)}_j + \gamma_{03}\texttt{(rural)}_j + \gamma_{04}\texttt{(suburb)}_j \\ &\quad + U_{0j} + R_{ij} \end{aligned}$$

Marginal model : $\texttt{(science)}_{ij} \sim \mathcal{N}(\mu_{ij}, (\tau_0^2 + \sigma^2))$ where

$$\begin{aligned} \mu_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\ &\quad + \gamma_{40}\texttt{(hours\_TV)}_{ij} + \gamma_{5j}\texttt{(hours\_computer\_games)}_{ij} \\ &\quad \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(isolated)}_j + \gamma_{03}\texttt{(rural)}_j + \gamma_{04}\texttt{(suburb)}_j \end{aligned}$$

(p) Hierarchical model :

Level 1 :

$$\begin{aligned}
\texttt{(science)}_{ij} &= \beta_{0j} + \beta_{1j}\texttt{(grpCmath)}_{ij} + \beta_{2j}\texttt{(gender)}_{ij} + \beta_{3j}\texttt{(grade)}_{ij} \\
&\quad + \beta_{4j}\texttt{(hours_TV)}_{ij} + \beta_{5j}\texttt{(hours_computer_games)}_{ij} + R_{ij}
\end{aligned}$$

where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2 :

$$\begin{aligned}
\beta_{0j} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_{j} + \gamma_{02}\texttt{(isolated)}_{j} + \gamma_{03}\texttt{(rural)}_{j} \\
&\quad + \gamma_{04}\texttt{(suburb)}_{j} + U_{0j} \\
\beta_{1j} &= \gamma_{10} + U_{1j} \\
\beta_{2j} &= \gamma_{20} \\
\beta_{3j} &= \gamma_{30} \\
\beta_{4j} &= \gamma_{40} \\
\beta_{5j} &= \gamma_{5j}
\end{aligned}$$

where

$$\begin{pmatrix} U_{0j} \\ U_{1j} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_0^2 & \tau_{01} \\ \tau_{01} & \tau_1^2 \end{pmatrix} \right) \qquad i.i.d.$$

and independent of R_{ij}.

Linear mixed model :

$$\begin{aligned}
\texttt{(science)}_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\
&\quad + \gamma_{40}\texttt{(hours_TV)}_{ij} + \gamma_{5j}\texttt{(hours_computer_games)}_{ij} \\
&\quad + \gamma_{01}\texttt{(grpMmath)}_{j} + \gamma_{02}\texttt{(isolated)}_{j} + \gamma_{03}\texttt{(rural)}_{j} + \gamma_{04}\texttt{(suburb)}_{j} \\
&\quad + U_{0j} + U_{1j}\texttt{(grpCmath)}_{ij} + R_{ij}
\end{aligned}$$

Marginal model : $\texttt{(science)}_{ij} \sim \mathcal{N}(\mu_{ij}, \mathrm{var}(Y_{ij}))$ where

$$\begin{aligned}
\mu_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\
&\quad + \gamma_{40}\texttt{(hours_TV)}_{ij} + \gamma_{5j}\texttt{(hours_computer_games)}_{ij} \\
&\quad + \gamma_{01}\texttt{(grpMmath)}_{j} + \gamma_{02}\texttt{(isolated)}_{j} + \gamma_{03}\texttt{(rural)}_{j} + \gamma_{04}\texttt{(suburb)}_{j}
\end{aligned}$$

$$\mathrm{var}(Y_{ij}) = \tau_0^2 + 2\tau_{12}\texttt{(grpCmath)}_{ij} + \tau_1^2\texttt{(grpCmath)}_{ij}^2 + \sigma^2$$

(q) Hierarchical model :

Level 1 :

$$\begin{aligned}
\texttt{(science)}_{ij} &= \beta_{0j} + \beta_{1j}\texttt{(grpCmath)}_{ij} + \beta_{2j}\texttt{(gender)}_{ij} + \beta_{3j}\texttt{(grade)}_{ij} \\
&\quad + \beta_{4j}\texttt{(hours_TV)}_{ij} + \beta_{5j}\texttt{(hours_computer_games)}_{ij} + R_{ij}
\end{aligned}$$

where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2 :

$$\begin{aligned}
\beta_{0j} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(isolated)}_j + \gamma_{03}\texttt{(rural)}_j \\
&\quad + \gamma_{04}\texttt{(suburb)}_j + U_{0j} \\
\beta_{1j} &= \gamma_{10} \\
\beta_{2j} &= \gamma_{20} \\
\beta_{3j} &= \gamma_{30} \\
\beta_{4j} &= \gamma_{40} + U_{4j} \\
\beta_{5j} &= \gamma_{5j}
\end{aligned}$$

where

$$\begin{pmatrix} U_{0j} \\ U_{4j} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_0^2 & \tau_{04} \\ \tau_{04} & \tau_4^2 \end{pmatrix} \right) \qquad i.i.d.$$

and independent of R_{ij}.

Linear mixed model :

$$\begin{aligned}
\texttt{(science)}_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\
&\quad + \gamma_{40}\texttt{(hours_TV)}_{ij} + \gamma_{5j}\texttt{(hours_computer_games)}_{ij} \\
&\quad + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(isolated)}_j + \gamma_{03}\texttt{(rural)}_j + \gamma_{04}\texttt{(suburb)}_j \\
&\quad + U_{0j} + U_{4j}\texttt{(hours_TV)}_{ij} + R_{ij}
\end{aligned}$$

Marginal model : $\texttt{(science)}_{ij} \sim \mathcal{N}(\mu_{ij}, \mathrm{var}(Y_{ij}))$ where

$$\begin{aligned}
\mu_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\
&\quad + \gamma_{40}\texttt{(hours_TV)}_{ij} + \gamma_{5j}\texttt{(hours_computer_games)}_{ij} \\
&\quad + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(isolated)}_j + \gamma_{03}\texttt{(rural)}_j + \gamma_{04}\texttt{(suburb)}_j
\end{aligned}$$

$$\mathrm{var}(Y_{ij}) = \tau_0^2 + 2\tau_{04}\texttt{(hours\_TV)}_{ij} + \tau_4^2\texttt{(hours\_TV)}_{ij}^2 + \sigma^2$$

(r) Hierarchical model :

Level 1 :

$$\begin{aligned}
\texttt{(science)}_{ij} &= \beta_{0j} + \beta_{1j}\texttt{(grpCmath)}_{ij} + \beta_{2j}\texttt{(gender)}_{ij} + \beta_{3j}\texttt{(grade)}_{ij} \\
&\quad + \beta_{4j}\texttt{(hours_TV)}_{ij} + \beta_{5j}\texttt{(hours_computer_games)}_{ij} + R_{ij}
\end{aligned}$$

where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2 :

$$\begin{aligned}
\beta_{0j} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(isolated)}_j + \gamma_{03}\texttt{(rural)}_j \\
&\quad + \gamma_{04}\texttt{(suburb)}_j + U_{0j} \\
\beta_{1j} &= \gamma_{10} \\
\beta_{2j} &= \gamma_{20} \\
\beta_{3j} &= \gamma_{30} \\
\beta_{4j} &= \gamma_{40} \\
\beta_{5j} &= \gamma_{5j} + U_{5j}
\end{aligned}$$

where

$$\begin{pmatrix} U_{0j} \\ U_{5j} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_0^2 & \tau_{05} \\ \tau_{05} & \tau_4^2 \end{pmatrix} \right) \qquad i.i.d.$$

and independent of R_{ij}.

Linear mixed model :

$$\begin{aligned}
\texttt{(science)}_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\
&\quad + \gamma_{40}\texttt{(hours_TV)}_{ij} + \gamma_{5j}\texttt{(hours_computer_games)}_{ij} \\
&\quad + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(isolated)}_j + \gamma_{03}\texttt{(rural)}_j + \gamma_{04}\texttt{(suburb)}_j \\
&\quad + U_{0j} + U_{5j}\texttt{(hours_computer_games)}_{ij} + R_{ij}
\end{aligned}$$

Marginal model : $\texttt{(science)}_{ij} \sim \mathcal{N}(\mu_{ij}, \mathrm{var}(Y_{ij}))$ where

$$\begin{aligned}
\mu_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\
&\quad + \gamma_{40}\texttt{(hours_TV)}_{ij} + \gamma_{5j}\texttt{(hours_computer_games)}_{ij} \\
&\quad + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(isolated)}_j + \gamma_{03}\texttt{(rural)}_j + \gamma_{04}\texttt{(suburb)}_j
\end{aligned}$$

$$\mathrm{var}(Y_{ij}) = \tau_0^2 + 2\tau_{05}\texttt{(hours\_computer\_games)}_{ij} + \tau_5^2\texttt{(hours\_computer\_games)}_{ij}^2 + \sigma^2$$

(s) Hierarchical model :

Level 1 :

$$
\begin{aligned}
\texttt{(science)}_{ij} &= \beta_{0j} + \beta_{1j}\texttt{(grpCmath)}_{ij} + \beta_{2j}\texttt{(gender)}_{ij} + \beta_{3j}\texttt{(grade)}_{ij} \\
&\quad + \beta_{4j}\texttt{(hours\_TV)}_{ij} + \beta_{5j}\texttt{(hours\_computer\_games)}_{ij} + R_{ij}
\end{aligned}
$$

where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2 :

$$
\begin{aligned}
\beta_{0j} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_{j} + \gamma_{02}\texttt{(isolated)}_{j} + \gamma_{03}\texttt{(rural)}_{j} \\
&\quad + \gamma_{04}\texttt{(suburb)}_{j} + U_{0j} \\
\beta_{1j} &= \gamma_{10} + \gamma_{11}\texttt{(grpMmath)}_{j} + \gamma_{12}\texttt{(isolated)}_{j} + \gamma_{13}\texttt{(rural)}_{j} \\
&\quad + \gamma_{14}\texttt{(suburb)}_{j} + U_{1j} \\
\beta_{2j} &= \gamma_{20} \\
\beta_{3j} &= \gamma_{30} \\
\beta_{4j} &= \gamma_{40} \\
\beta_{5j} &= \gamma_{5j}
\end{aligned}
$$

where

$$
\begin{pmatrix} U_{0j} \\ U_{1j} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_0^2 & \tau_{01} \\ \tau_{01} & \tau_1^2 \end{pmatrix} \right) \qquad i.i.d.
$$

and independent of R_{ij}.

Linear mixed model :

$$
\begin{aligned}
\texttt{(science)}_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\
&\quad + \gamma_{40}\texttt{(hours\_TV)}_{ij} + \gamma_{5j}\texttt{(hours\_computer\_games)}_{ij} \\
&\quad + \gamma_{01}\texttt{(grpMmath)}_{j} + \gamma_{02}\texttt{(isolated)}_{j} + \gamma_{03}\texttt{(rural)}_{j} + \gamma_{04}\texttt{(suburb)}_{j} \\
&\quad + \gamma_{11}\texttt{(grpMmath)}_{j}\texttt{(grpCmath)}_{ij} + \gamma_{12}\texttt{(isolated)}_{j}\texttt{(grpCmath)}_{ij} \\
&\quad + \gamma_{13}\texttt{(rural)}_{j}\texttt{(grpCmath)}_{ij} + \gamma_{14}\texttt{(suburb)}_{j}\texttt{(grpCmath)}_{ij} \\
&\quad + U_{0j} + U_{1j}\texttt{(grpCmath)}_{ij} + R_{ij}
\end{aligned}
$$

Marginal model : $\texttt{(science)}_{ij} \sim \mathcal{N}(\mu_{ij}, \mathrm{var}(Y_{ij}))$ where

$$
\begin{aligned}
\mu_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\
&\quad + \gamma_{40}\texttt{(hours\_TV)}_{ij} + \gamma_{5j}\texttt{(hours\_computer\_games)}_{ij} \\
&\quad + \gamma_{01}\texttt{(grpMmath)}_{j} + \gamma_{02}\texttt{(isolated)}_{j} + \gamma_{03}\texttt{(rural)}_{j} + \gamma_{04}\texttt{(suburb)}_{j} \\
&\quad + \gamma_{11}\texttt{(grpMmath)}_{j}\texttt{(grpCmath)}_{ij} + \gamma_{12}\texttt{(isolated)}_{j}\texttt{(grpCmath)}_{ij} \\
&\quad + \gamma_{13}\texttt{(rural)}_{j}\texttt{(grpCmath)}_{ij} + \gamma_{14}\texttt{(suburb)}_{j}\texttt{(grpCmath)}_{ij}
\end{aligned}
$$

$$
\mathrm{var}(Y_{ij}) = \tau_0^2 + 2\tau_{01}\texttt{(grpCmath)}_{ij} + \tau_1^2\texttt{(grpCmath)}_{ij}^2 + \sigma^2
$$

(t) Hierarchical model :

Level 1 :

$$\begin{aligned}\texttt{(science)}_{ij} &= \beta_{0j} + \beta_{1j}\texttt{(grpCmath)}_{ij} + \beta_{2j}\texttt{(gender)}_{ij} + \beta_{3j}\texttt{(grade)}_{ij}\\ &\quad +\beta_{4j}\texttt{(hours\_TV)}_{ij} + \beta_{5j}\texttt{(hours\_computer\_games)}_{ij} + R_{ij}\end{aligned}$$

where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2 :

$$\begin{aligned}\beta_{0j} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_{j} + \gamma_{02}\texttt{(isolated)}_{j} + \gamma_{03}\texttt{(rural)}_{j}\\ &\quad +\gamma_{04}\texttt{(suburb)}_{j} + U_{0j}\\ \beta_{1j} &= \gamma_{10} + \gamma_{11}\texttt{(grpMmath)}_{j} + \gamma_{12}\texttt{(isolated)}_{j} + \gamma_{13}\texttt{(rural)}_{j}\\ &\quad +\gamma_{14}\texttt{(suburb)}_{j} + \gamma_{15}\texttt{(shortages)}_{j} + U_{1j}\\ \beta_{2j} &= \gamma_{20}\\ \beta_{3j} &= \gamma_{30}\\ \beta_{4j} &= \gamma_{40}\\ \beta_{5j} &= \gamma_{5j}\end{aligned}$$

where

$$\begin{pmatrix} U_{0j} \\ U_{1j} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_0^2 & \tau_{01} \\ \tau_{01} & \tau_1^2 \end{pmatrix}\right) \qquad i.i.d.$$

and independent of R_{ij}.

Linear mixed model :

$$\begin{aligned}\texttt{(science)}_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij}\\ &\quad +\gamma_{40}\texttt{(hours\_TV)}_{ij} + \gamma_{5j}\texttt{(hours\_computer\_games)}_{ij}\\ &\quad +\gamma_{01}\texttt{(grpMmath)}_{j} + \gamma_{02}\texttt{(isolated)}_{j} + \gamma_{03}\texttt{(rural)}_{j} + \gamma_{04}\texttt{(suburb)}_{j}\\ &\quad +\gamma_{11}\texttt{(grpMmath)}_{j}\texttt{(grpCmath)}_{ij} + \gamma_{12}\texttt{(isolated)}_{j}\texttt{(grpCmath)}_{ij}\\ &\quad +\gamma_{13}\texttt{(rural)}_{j}\texttt{(grpCmath)}_{ij} + \gamma_{14}\texttt{(suburb)}_{j}\texttt{(grpCmath)}_{ij}\\ &\quad +\gamma_{15}\texttt{(shortages)}_{j}\texttt{(grpCmath)}_{ij}\\ &\quad +U_{0j} + U_{1j}\texttt{(grpCmath)}_{ij} + R_{ij}\end{aligned}$$

Marginal model : $\texttt{(science)}_{ij} \sim \mathcal{N}(\mu_{ij}, \mathrm{var}(Y_{ij}))$ where

$$\begin{aligned}\mu_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij}\\ &\quad +\gamma_{40}\texttt{(hours\_TV)}_{ij} + \gamma_{5j}\texttt{(hours\_computer\_games)}_{ij}\\ &\quad +\gamma_{01}\texttt{(grpMmath)}_{j} + \gamma_{02}\texttt{(isolated)}_{j} + \gamma_{03}\texttt{(rural)}_{j} + \gamma_{04}\texttt{(suburb)}_{j}\\ &\quad +\gamma_{11}\texttt{(grpMmath)}_{j}\texttt{(grpCmath)}_{ij} + \gamma_{12}\texttt{(isolated)}_{j}\texttt{(grpCmath)}_{ij}\\ &\quad +\gamma_{13}\texttt{(rural)}_{j}\texttt{(grpCmath)}_{ij} + \gamma_{14}\texttt{(suburb)}_{j}\texttt{(grpCmath)}_{ij}\\ &\quad +\gamma_{15}\texttt{(shortages)}_{j}\texttt{(grpCmath)}_{ij}\end{aligned}$$

$$\mathrm{var}(Y_{ij}) = \tau_0^2 + 2\tau_{01}\texttt{(grpCmath)}_{ij} + \tau_1^2\texttt{(grpCmath)}_{ij}^2 + \sigma^2$$

(u) Hierarchical model :

Level 1 :

$$
\begin{aligned}
\texttt{(science)}_{ij} &= \beta_{0j} + \beta_{1j}\texttt{(grpCmath)}_{ij} + \beta_{2j}\texttt{(gender)}_{ij} + \beta_{3j}\texttt{(grade)}_{ij} \\
&\quad + \beta_{4j}\texttt{(hours\_TV)}_{ij} + \beta_{5j}\texttt{(hours\_computer\_games)}_{ij} + R_{ij}
\end{aligned}
$$

where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2 :

$$
\begin{aligned}
\beta_{0j} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(isolated)}_j + \gamma_{03}\texttt{(rural)}_j \\
&\quad + \gamma_{04}\texttt{(suburb)}_j + U_{0j} \\
\beta_{1j} &= \gamma_{10} + \gamma_{11}\texttt{(grpMmath)}_j + \gamma_{12}\texttt{(isolated)}_j + \gamma_{13}\texttt{(rural)}_j \\
&\quad + \gamma_{14}\texttt{(suburb)}_j + U_{1j} \\
\beta_{2j} &= \gamma_{20} \\
\beta_{3j} &= \gamma_{30} \\
\beta_{4j} &= \gamma_{40} + U_{4j} \\
\beta_{5j} &= \gamma_{5j}
\end{aligned}
$$

where

$$
\begin{pmatrix} U_{0j} \\ U_{1j} \\ U_{4j} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_0^2 & \tau_{01} & \tau_{04} \\ \tau_{01} & \tau_1^2 & \tau_{14} \\ \tau_{04} & \tau_{14} & \tau_4^2 \end{pmatrix} \right) \qquad i.i.d.
$$

and independent of R_{ij}.

Linear mixed model :

$$
\begin{aligned}
\texttt{(science)}_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\
&\quad + \gamma_{40}\texttt{(hours\_TV)}_{ij} + \gamma_{5j}\texttt{(hours\_computer\_games)}_{ij} \\
&\quad + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(isolated)}_j + \gamma_{03}\texttt{(rural)}_j + \gamma_{04}\texttt{(suburb)}_j \\
&\quad + \gamma_{11}\texttt{(grpMmath)}_j\texttt{(grpCmath)}_{ij} + \gamma_{12}\texttt{(isolated)}_j\texttt{(grpCmath)}_{ij} \\
&\quad + \gamma_{13}\texttt{(rural)}_j\texttt{(grpCmath)}_{ij} + \gamma_{14}\texttt{(suburb)}_j\texttt{(grpCmath)}_{ij} \\
&\quad + U_{0j} + U_{1j}\texttt{(grpCmath)}_{ij} + U_{4j}\texttt{(hours\_TV)}_{ij} + R_{ij}
\end{aligned}
$$

Marginal model : $\texttt{(science)}_{ij} \sim \mathcal{N}(\mu_{ij}, \mathrm{var}(Y_{ij}))$ where

$$\begin{aligned}
\mu_{ij} &= \gamma_{00} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} \\
&\quad + \gamma_{40}\texttt{(hours_TV)}_{ij} + \gamma_{5j}\texttt{(hours_computer_games)}_{ij} \\
&\quad + \gamma_{01}\texttt{(grpMmath)}_{j} + \gamma_{02}\texttt{(isolated)}_{j} + \gamma_{03}\texttt{(rural)}_{j} + \gamma_{04}\texttt{(suburb)}_{j} \\
&\quad + \gamma_{11}\texttt{(grpMmath)}_{j}\texttt{(grpCmath)}_{ij} + \gamma_{12}\texttt{(isolated)}_{j}\texttt{(grpCmath)}_{ij} \\
&\quad + \gamma_{13}\texttt{(rural)}_{j}\texttt{(grpCmath)}_{ij} + \gamma_{14}\texttt{(suburb)}_{j}\texttt{(grpCmath)}_{ij}
\end{aligned}$$

$$\begin{aligned}
\mathrm{var}(Y_{ij}) &= \tau_0^2 + 2\tau_{01}\texttt{(grpCmath)}_{ij} + 2\tau_{04}\texttt{(hours_TV)}_{ij} + 2\tau_{14}\texttt{(grpCmath)}_{ij}\texttt{(hours_TV)}_{ij} \\
&\quad + \tau_1^2\texttt{(grpCmath)}^2_{ij} + \tau_4^2\texttt{(hours_TV)}^2_{ij} + \sigma^2
\end{aligned}$$

(v) Any model that you thought was "best".

(a) Summary Table (10 points)

	# of		Fixed Effects			Random Effects				Fit statistics	
	estimated					Between		Within			
Model	params	"Effect"	γ	Estimate	SE	$\hat{\tau}^2$	SE	$\hat{\sigma}^2$	SE	-2 loglike	AIC
(j)	7	intercept	$\hat{\gamma}_{00}$	10.3448	0.5528	3.7542	0.5884	51.7541	0.8782	48363.7	48377.7
		`grpMmath`	$\hat{\gamma}_{01}$	0.9002	0.0396						
		`grpCmath`	$\hat{\gamma}_{10}$	0.5528	0.0100						
		`grade`	$\hat{\gamma}_{20}$	0.9253	0.1953						
	`gender`	boys	$\hat{\gamma}_{30}$	1.1154	0.1720						
		girls		0	.						
(j) Random int w/ grade dummy coded	7	intercept	$\hat{\gamma}_{00}$	14.0460	5.9717	Everything else is the same					
	`grade`	3rd	$\hat{\gamma}_{20}$	-0.9253	0.1953						
		4th		0.0000	.						
(n)	9	Intercept	γ_{00}	16.0045	5.9410	3.6654	0.5784	51.6515	0.8765	48347.2	48365.2
		grpCmath	γ_{10}	0.5510	0.0100						
	gender	boy	γ_{20}	-0.9194	0.1952						
		girl		0	.						
	grade	3rd	γ_{30}	1.2089	0.1733						
		4th		0	.						
		hoursTV	γ_{40}	-0.0885	0.0746						
		hours computer	γ_{50}	-0.2795	0.0786						
		grpMmath	γ_{01}	0.8924	0.03923						
(o)	12	Intercept	γ_{00}	15.93	5.86	3.38	0.55	51.65	0.88	48338.6	48362.6
		grpCmath	γ_{10}	0.55	0.01						
	gender	boy	γ_{20}	1.21	0.17						
		girl		0	.						
	grade	3rd	γ_{30}	-0.93	0.20						
		4th		0	.						
		hours TV	γ_{40}	-0.09	0.07						
		hours computer	γ_{50}	-0.28	0.08						
		grpMmath	γ_{01}	0.89	0.04						
	type	isolated	γ_{02}	4.32	2.04						
		rural	γ_{03}	1.08	0.51						
		suburb	γ_{04}	0.09	0.40						
		urban		0	.						

	# estimated		Fixed Effects			Random Effects		Fit statistics	
Model	parameters	"Effect"	γ	Estimate	SE	estimate	SE	-2 loglike	AIC
(p)	14	Intercept	γ_{00}	8.7405	5.7810	$\tau_0^2 = 3.4404$	.5604	48306.8	48334.8
		grpCmath	γ_{10}	0.5574	0.0131	$\tau_{10} = 0.0640$	.0343		
	gender	boy	γ_{20}	1.2575	0.1726	$\tau_2^2 = 0.0091$	.0026		
		girl		0	.	$\sigma^2 = 50.9044$	.8724		
	grade	3rd	γ_{30}	-0.8962	0.1951				
		4th		0	.				
		hours TV	γ_{40}	-0.0832	0.0743				
		hours computer	γ_{50}	-0.2743	0.0784				
		grpMmath	γ_{01}	0.9390	0.0384				
	type	isolated	γ_{02}	3.9508	2.0002				
		rural	γ_{03}	0.9247	0.5010				
		suburb	γ_{04}	-0.0220	0.3967				
		urban		0	.				
(q)	14	Intercept	γ_{00}	15.0227	5.8080	$\tau_0^2 = 3.6886$	1.5034	48334.1	48362.1
		grpCmath	γ_{10}	0.5512	0.0100	$\tau_{40} = -0.3133$	0.3705		
	gender	boy	γ_{20}	1.1995	0.1733	$\tau_4^2 = 0.1677$	0.1102		
		girl		0	.	$\sigma^2 = 51.4261$	0.8812		
	grade	3rd	γ_{30}	-0.9188	0.1952				
		4th		0	.				
		hours TV	γ_{40}	-0.0886	0.0827				
		hours computer	γ_{50}	-0.2792	0.0787				
		grpMmath	γ_{01}	0.8976	0.0386				
	type	isolated	γ_{02}	4.1308	1.9976				
		rural	γ_{03}	1.0837	0.5050				
		suburb	γ_{04}	0.0285	0.3998				
		urban		0	.				

Model	# estimated parameters	"Effect"	Fixed Effects γ	Fixed Effects Estimate	Fixed Effects SE	Random Effects estimate	Random Effects SE	Fit statistics -2 loglike	Fit statistics AIC
(r)	14	Intercept	γ_{00}	15.5622	5.8151	$\tau_0^2 = 4.0594$	1.0135	48337.7	48365.7
		grpCmath	γ_{10}	0.5509	0.0100	$\tau_{50} = -0.1943$	0.2739		
	gender	boy	γ_{20}	1.2122	0.1733	$\tau_5^2 = 0.0179$	0.0973		
		girl		0	.	$\sigma^2 = 51.6365$	0.8844		
	grade	3rd	γ_{30}	-0.9304	0.1951				
		4th		0	.				
		hours TV	γ_{40}	-0.0889	0.0747				
		hours computer	γ_{50}	-0.2816	0.0794				
		grpMmath	γ_{01}	0.8937	0.0386				
	type	isolated	γ_{02}	4.3036	2.0419				
		rural	γ_{03}	1.1341	0.5066				
		suburb	γ_{04}	0.1302	0.4007				
		urban		0	.				
(s)	18	Intercept	γ_{00}	15.8565	5.8623	$\tau_0^2 = 3.3974$	0.5476	48255.4	48291.4
		grpCmath	γ_{10}	3.3033	0.3548	$\tau_{10} = 0.0444$	0.0238		
	gender	boy	γ_{20}	1.2766	0.1724	$\tau_1^2 = 0.0022$	0.0017		
		girl		0	.	$\sigma^2 = 50.9033$	0.8717		
	grade	3rd	γ_{30}	-0.8738	0.1941				
		4th		0	.				
		hours TV	γ_{40}	-0.0914	0.0741				
		hours computer	γ_{50}	-0.2557	0.0782				
		grpMmath	γ_{01}	0.8913	0.0389				
	type	isolated	γ_{02}	4.3284	2.0471				
		rural	γ_{03}	1.0834	0.5079				
		suburb	γ_{04}	0.0789	0.4032				
		urban	0 .						
	Cross-level	grpCmath*grpMmath	γ_{11}	-0.0183	0.0024				
		grpCmath*isolated	γ_{12}	0.0987	0.1053				
		grpCmath*rural	γ_{13}	0.0558	0.0315				
		grpCmath*suburb	γ_{14}	0.0344	0.0233				
		grpCmath*urban		0	.				

Model	# estimated parameters	"Effect"	Fixed Effects γ	Estimate	SE	Random Effects estimate	SE	Fit statistics -2 loglike	AIC
(t)	19	Intercept	γ_{00}	15.8529	5.8623	$\tau_0^2 = 3.3975$	0.5476	48255.1	48293.1
		grpCmath	γ_{10}	3.3528	0.3656	$\tau_{10} = 0.0439$	0.0238		
	gender	boy	γ_{20}	-0.8728	0.1941	$\tau_1^2 = 0.0023$	0.0017		
		girl		0	.	$\sigma^2 = 50.8984$	0.8715		
	grade	3rd	γ_{30}	1.2774	0.1724				
		4th		0	.				
		hours TV	γ_{40}	-0.0915	0.0741				
		hours computer	γ_{50}	-0.2555	0.0782				
		grpMmath	γ_{01}	0.8913	0.0389				
	type	isolated	γ_{02}	4.3287	2.0471				
		rural	γ_{03}	1.0830	0.5079				
		suburb	γ_{04}	0.0794	0.4032				
		urban	0 0 .						
	Cross-level	grpCmath*grpMmath	γ_{11}	-0.0186	0.0024				
		grpCmath*isolated	γ_{12}	0.0944	0.1057				
		grpCmath*rural	γ_{13}	0.0575	0.0316				
		grpCmath*suburb	γ_{14}	0.0348	0.0234				
		grpCmath*urban		0	.				
		grpCmath*shortages	γ_{15}	-0.0071	0.0123				
(u)	22	Intercept	γ_{00}			$\tau_0^2 = 3.1692$	1.4122	48247.3	48289.3
		grpCmath	γ_{10}			$\tau_{10} = 0.0955$	0.0370		
	gender	boy	γ_{20}			$\tau_1^2 = 0.0023$	0.0017		
		girl				$\tau_{04} = -0.1607$	0.3438		
	grade	3rd	γ_{30}			$\tau_{14} = -0.0167$	0.0099		
		4th				$\tau_4^2 = 0.1289$	0.1023		
		hours TV	γ_{40}			$\sigma^2 = 50.7218$	0.8766		
		hours computer	γ_{50}						
		grpMmath	γ_{01}						
	type	isolated	γ_{02}						
		rural	γ_{03}						
		suburb	γ_{04}						
		urban	0						

etc.

	# estimated		Fixed Effects			Random Effects		Fit statistics	
Model	parameters	"Effect"	γ	Estimate	SE	estimate	SE	-2 loglike	AIC
(v)	14	Intercept	γ_{00}	14.5723	5.8749	$\tau_0^2 = 3.4335$	0.5521	48270.7	48298.7
		grpCmath	γ_{10}	3.2725	0.3520	$\tau_{10} = 0.04422$	0.02455		
	grade	3rd	γ_{30}	-0.8812	0.1943	$\tau_1^2 = .002514$	0.001749		
		4th		0	.	$\sigma^2 = 50.9873$	0.8733		
	gender	boy	γ_{20}	1.2081	0.1714	Random intercept & slope for grpCmath			
		girl		0	.				
		hours TV	γ_{40}	-0.1440	0.07249				
		grpMmath	γ_{01}	0.8981	0.03903				
	type	isolated	γ_{02}	3.9784	2.0129				
		rural	γ_{03}	0.9653	0.5036				
		suburb	γ_{04}	-0.03561	0.3989				
		urban		0	.				
	Cross-level	grpCmath*grpMmath	γ_{11}	-0.01792	0.002320				

3. (5 points) Does is appear that you need a random slope for GRPCMATH, HOURS_TV, and/or HOURS_COMPUTER_GAMES? Explain your reasoning.

   Things that could be said:

   - It appears that we need a random slope for GRPCMATH because the $\hat{\tau}_1^2$ tends to be "large" relative to it's standard error; whereas, the $\hat{\tau}_1^2$ for random slopes for the others are "small" relative to their standard errors.
   - Among the models that are the same except for which variable has a random slope (i.e., (m), (n) & (o)), model (m) with a random slope for GRPCMATH fits the best (can look at -2LnLike or AIC.
   - The following figures (at the end of the answer key) show that only GRPCMATH appears to have different slopes over groups than the other possible explanatory (micro) variables (not expected, but helpful to understand what's going on).
   - Anything that makes sense.

4. The variable GRPMMATH appears to be a useful predictors of the slope for GRPCMATH (See model (t)). The cross-level interactions between GRPCMATH & type of community and between GRPCMATH & don't appear "significant".

5. (5 points) My favorite model ...

   Note that I found in model (v), which is my favorite model of those that we fit, that γ_{04}, which was the fixed effect parameter for $\texttt{(suburb)}_j$ was not significantly different from 0, so I recoded the data as

$$\texttt{(isolated)}_j = \begin{cases} 1 & \text{for isolated school} \\ 0 & \text{otherwise} \end{cases} \qquad \texttt{(rural)}_j = \begin{cases} 1 & \text{for isolated school} \\ 0 & \text{otherwise} \end{cases}$$

   If a school was urban or suburban, then $\texttt{(isolated)}_j = \texttt{(rural)}_j = 0$.

Level 1:

$$\begin{aligned} \texttt{(science)}_{ij} &= \beta_{0j} + \beta_{1j}\texttt{(grpCmath)}_{ij} + \beta_{2j}\texttt{(gender)}_{ij} + \beta_{3j}\texttt{(grade)}_{ij} \\ &\quad + \beta_{4j}\texttt{(hours\_TV)}_{ij} + R_{ij} \end{aligned}$$

where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2 :

$$\begin{aligned}
\beta_{0j} &= \gamma_{00} + \gamma_{01}(\texttt{grpMmath})_j + \gamma_{02}(\texttt{isolated})_j + \gamma_{03}(\texttt{rural})_j + U_{0j} \\
\beta_{1j} &= \gamma_{10} + \gamma_{11}(\texttt{grpMmath})_j + U_{1j} \\
\beta_{2j} &= \gamma_{20} \\
\beta_{3j} &= \gamma_{30} \\
\beta_{4j} &= \gamma_{40}
\end{aligned}$$

where

$$\begin{pmatrix} U_{0j} \\ U_{4j} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_0^2 & \tau_{01} \\ \tau_{01} & \tau_1^2 \end{pmatrix}\right) \qquad i.i.d.$$

and independent of R_{ij}.

The parameter estimates are

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	idschool	3.4325	0.5518	6.22	<.0001
UN(2,1)	idschool	0.04390	0.02427	1.81	0.0705
UN(2,2)	idschool	0.002514	0.001749	1.44	0.0754
Residual		50.9871	0.8733	58.39	<.0001

Fixed Effects Parameter Estimates

Effect	gender	Students grade level	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept			14.6645	5.7815	143	2.54	0.0123
grpCmath			3.2725	0.3520	144	9.30	<.0001
grade		3	-0.8814	0.1943	6801	-4.54	<.0001
grade		4	0	.	.	.	.
gender	boy		1.2081	0.1714	6801	7.05	<.0001
gender	girl		0	.	.	.	.
hours_TV			-0.1440	0.07249	6801	-1.99	0.0470
grpMmath			0.8974	0.03820	6801	23.50	<.0001
community	(isolated)		3.9954	2.0066	6801	1.99	0.0465
community	(rural)		0.9805	0.4766	6801	2.06	0.0397
community	(urban & suburban)		0	.	.	.	.
grpCmath*grpMmath			-0.01792	0.002320	6801	-7.72	<.0001

Summary/Interpretation: We find higher science scores for boys in the 4th grade who have higher math scores relative to their peers, and those who don't watch a lot of TV. Also, we find higher science scores in schools that have high average math scores. Furthermore, students in schools have the higher science scores in isolated locations, followed by rural. There don't appear to be differences between students in schools in urban & sub-urban locations. Those with the lowest scores are from urban and suburban locations.

We can give a more detailed explanation and put the parameter estimates into the models. For example,

$$\widehat{(\texttt{science})}_{ij} = \beta_{0j} + \beta_{1j}(\texttt{grpCmath})_{ij} + 1.21(\texttt{gender})_{ij} - 0.88(\texttt{grade})_{ij} - 0.14(\texttt{hours\_TV})_{ij},$$

where the estimates of the random parameters are

$$\begin{aligned} \beta_{0j} &= 14.66 + 0.90(\texttt{grpMmath})_j + 4.00(\texttt{isolated})_j + 0.98(\texttt{rural})_j \\ \beta_{1j} &= 3.27 - 0.02(\texttt{grpMmath})_j. \end{aligned}$$

- Science scores for boys are 1.21 points higher than those for girls.
- 3rd graders have science scores that are $-.88$ points lower then 4th graders (or scores for 4th graders are 0.88 points higher).
- Science scores tend to be -.14 of a point lower for every hour of TV watched during the week.
- Differences exist between schools.
  - Interpretation of random intercept: Overall, students have science scores that are
    * 0.90 points higher at schools where average math scores are 1 point higher.
    * 4.00 points higher at isolated schools than at urban or sub-urban schools.
    * 0.98 points higher at rural schools than at urban or sub-urban schools.
    * The variance of the intercept over schools is 3.43 (standard deviation of 1.85)— this is how much unexplained variation there is between schools in terms of the overall level of science scores.
  - Interpretation of random slope: For a 1 unit increase in a student's math scores relative to their peers, the expected change in the student's science score is $3.27 - 0.02(\texttt{grpMmath})_j$. If the student goes to a school with an average math score that's 1 unit higher, then the change in science score would be $3.27 - .02 = 3.25$. In appears that if a student has a higher math score relative to their peers but that the school overall has lower average math scores, we expect this student to have higher science scores.

    This is an example where the micro and macro effects are in opposite directions. If we use student math score (raw) or overall mean centered we get the slope for math equal to $(3.2725 - 2.3751(\texttt{grpMmath})_j)\text{math}_{ij}$, which shows this opposite micro/macro effect even more clearly. This seems to me counter-intuitive— any suggestions or explanations?

> The unexplained variance between schools in terms of the effect of student math scores on science scores is .002 (i.e., standard deviation of the slopes is pretty small).

In the figures at the end of this answer key are plots of data. The plots of science scores versus potential micro level predictors suggest that we need a random intercept. The "flatness" of the lines in the plots for gender, hour TV and hours computer games look like there may not be effects for these or if there are they are relatively small. The figure for science scores versus math scores indicate that this may be a stronger effect (not positive slope) and need a random slope.

On the last two pages, are plots of a sample of individual schools by math scores with best fit regression line for that school drawn in. It looks like linear regression OK (within schools). Can also see various differences between schools.

These plots and re-coding data are things that we'll cover in computer labs to come...

6. (5 points) Where there any problems in estimating the new models or problems in the solutions found by SAS? If yes, what was/were the problem(s)?

   There was a problem: In the SAS/LOG for model (u):

```
NOTE: Estimated G matrix is not positive definite.
```

   The matrix $\hat{\boldsymbol{T}}$ of estimated τ's,

Effect	Intercept	grpCmath	hours TV
Intercept	3.1692	0.09551	−0.1607
grpCmath	0.0955	0.00231	−0.0167
hours TV	−0.1607	−0.0167	0.1289

   is not a "proper" covariance matrix. Note that:

   - If we turn this into a correlation matrix, we get a "bad" correlation (i.e.,

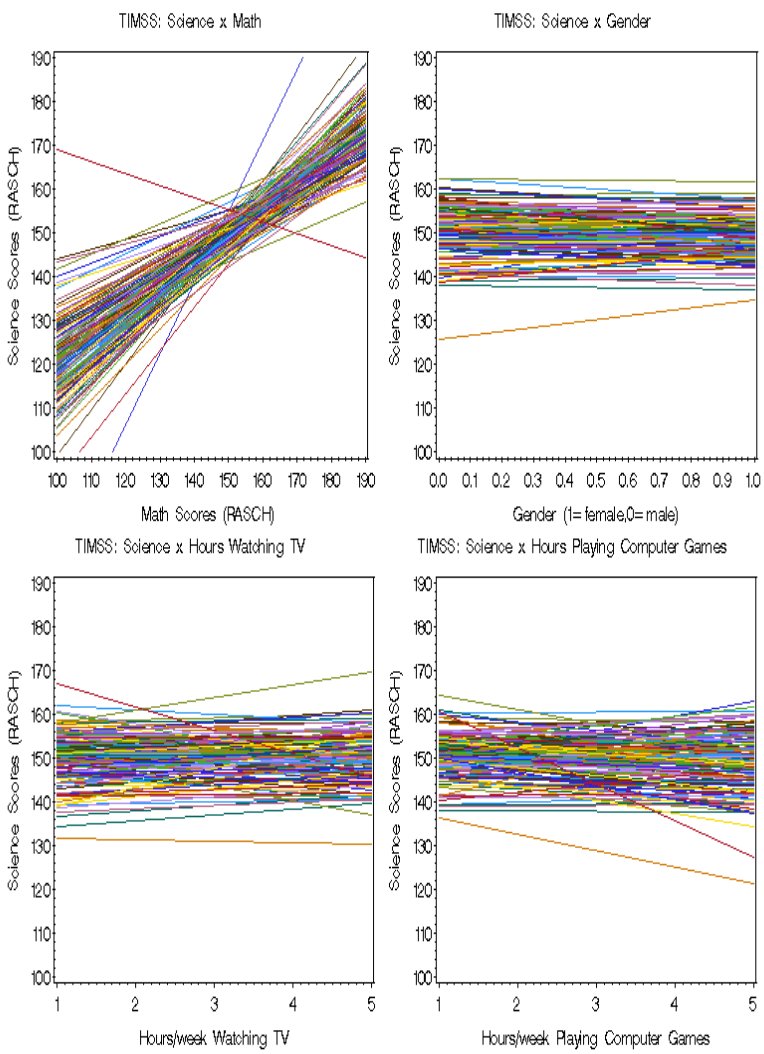
     r(intercept,grpCmath) $= 1.000$

Effect	Intercept	grpCmath	hours TV
Intercept	1.000	1.116	−0.2514
grpCmath	1.1161	1.0000	−0.9678
hours TV	−0.251	−0.9678	1.0000

   - If this is a good covariance matrix then $|\tau_{01}| \leq \sqrt{\tau_0^2}\sqrt{\tau_1^2}$, but we have

$$.0955 > \sqrt{3.1692}\sqrt{.00231} \longrightarrow .0955 > .0856 \quad \text{BAD}$$

   - Bad model for the data.


TIMSS: Science x Math
Science Scores (RASCH)
Math Scores (RASCH)
TIMSS: Science x Gender
Gender (1=female,0=male)
TIMSS: Science x Hours Watching TV
Hours/week Watching TV
TIMSS: Science x Hours Playing Computer Games
Hours/week Playing Computer Games

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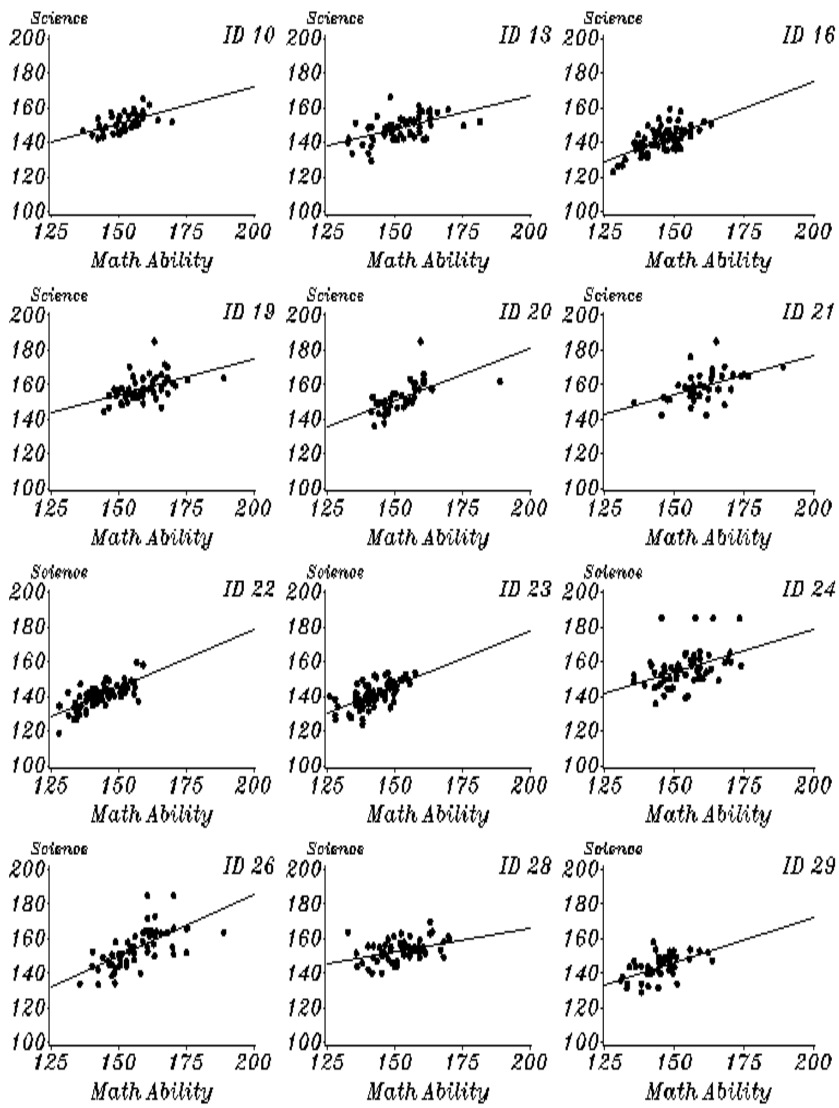
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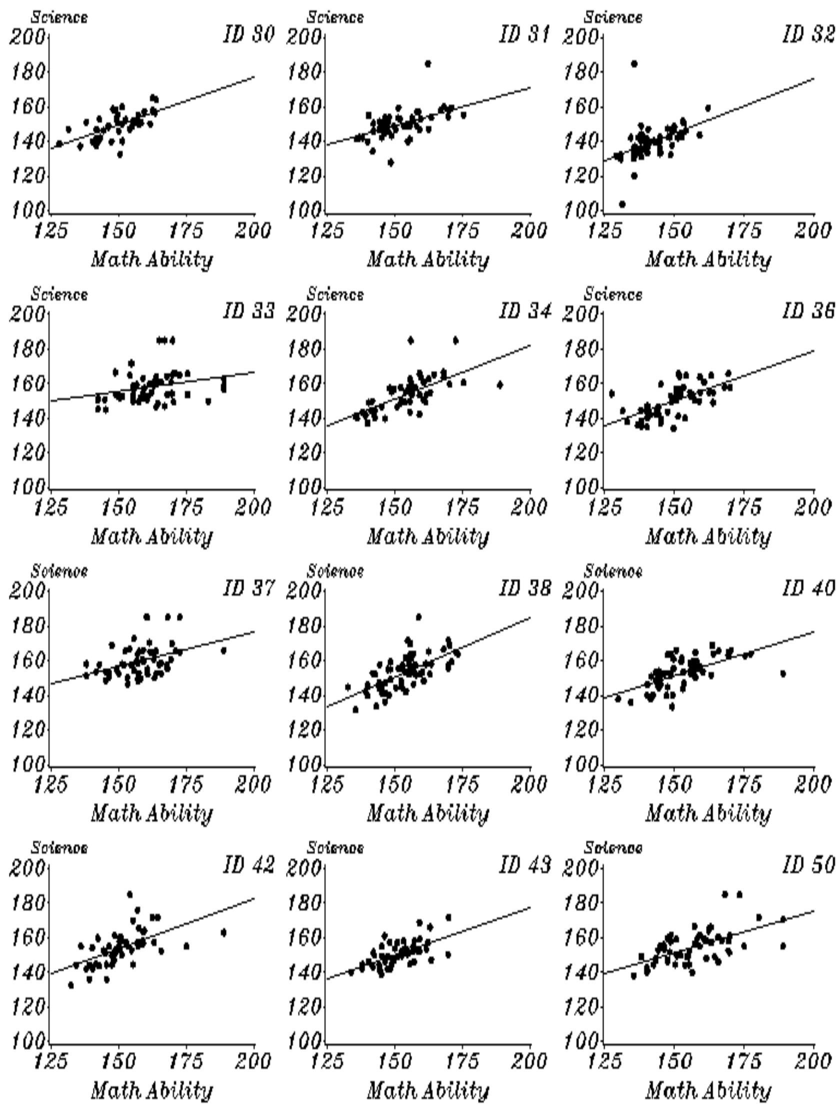
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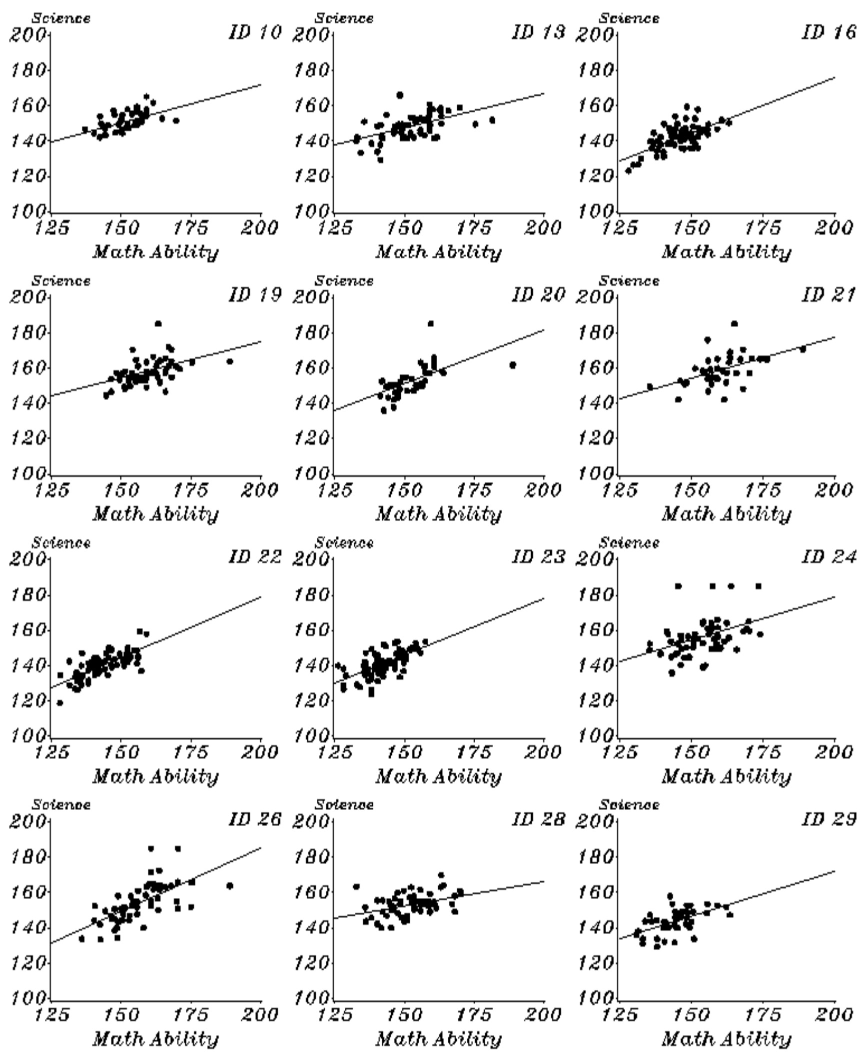
Science
Math Ability



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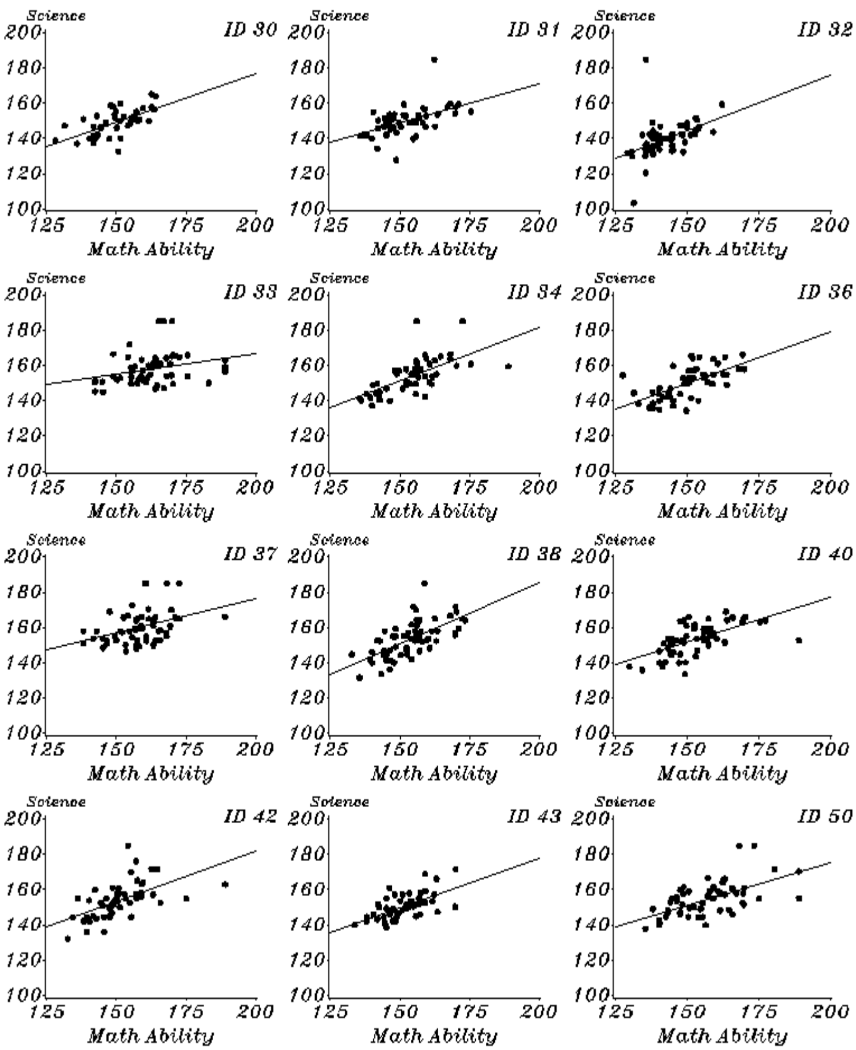
Math Ability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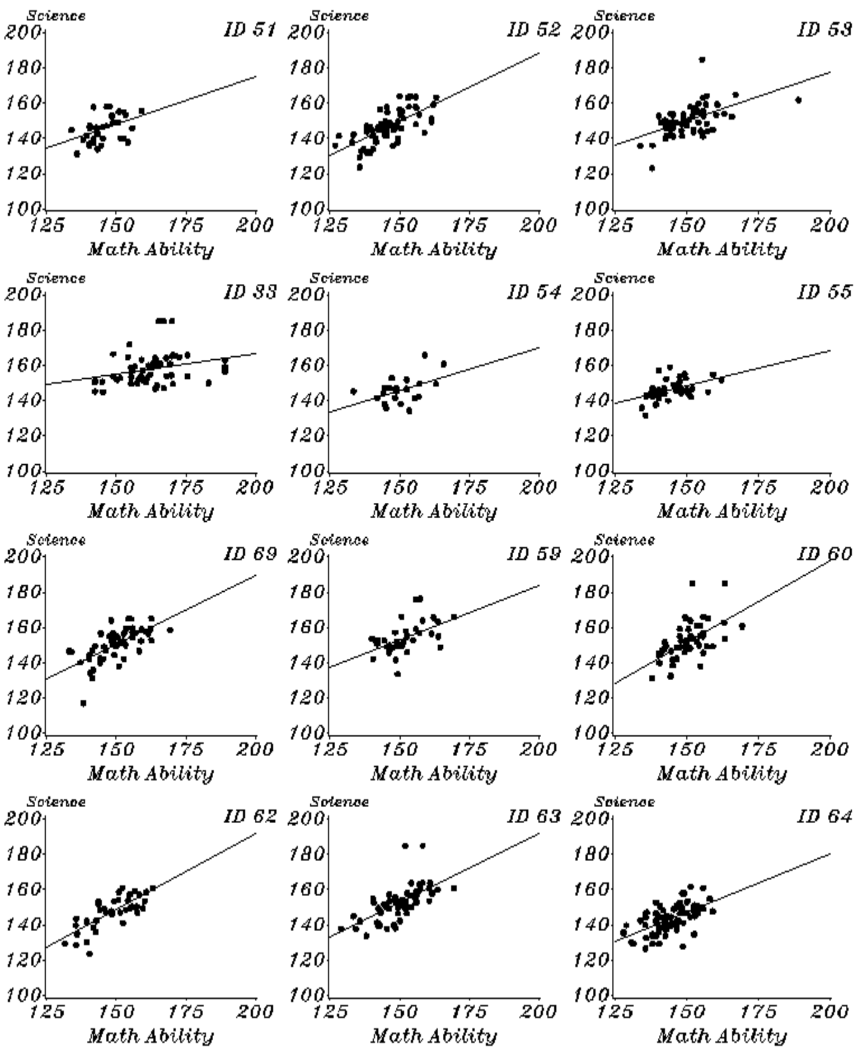
Math Ability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Math Ability