EdPsych/Psych/Stat 587
C.J. Anderson

SAS Homework # 3
Answer Key

1. (10 points) For models (a) – (j) fit to the USA TIMSS data, write out the equation of the model in each of the following ways:

(a) Fixed effects ANOVA

Hierarchical model :

Level 1: $(\texttt{science})_{ij} = \beta_{0j} + R_{ij}$ where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2: $\beta_{0j} = \gamma_{00} + \alpha_j$ or $\beta_{0j} = \mu + \alpha_j$
where $j = 1, \ldots, 146$. The α_j and γ_{00} (or μ) are fixed population parameters.

Linear mixed model : $(\texttt{science})_{ij} = \gamma_{00} + \alpha_j + R_{ij}$

Marginal model : $(\texttt{science})_{ij} \sim \mathcal{N}(\gamma_{00} + \alpha_j, \sigma^2)$

(b) ANCOVA with overall mean centered math, `OCmath`

Hierarchical model :

Level 1: $(\texttt{science})_{ij} = \beta_{0j} + \beta_{1j}(\texttt{OCmath})_{ij} + R_{ij}$
where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2:

$$\begin{aligned} \beta_{0j} &= \gamma_{00} + \alpha_j \\ \beta_{1j} &= \gamma_{10} \end{aligned}$$

The α_j, γ_{00} & γ_{10} are fixed population parameters.

Linear mixed model : $(\texttt{science})_{ij} = \gamma_{00} + \alpha_j + \gamma_{10}(\texttt{OCmath})_{ij} + R_{ij}$

Marginal model : $(\texttt{science})_{ij} \sim \mathcal{N}(\gamma_{00} + \alpha_j + \gamma_{10}(\texttt{OCmath})_{ij}, \sigma^2)$.

(c) Random Effects ANOVA

Hierarchical model: Level 1: $(\texttt{science})_{ij} = \beta_{0j} + R_{ij}$
where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2: $\beta_{0j} = \gamma_{00} + U_{0j}$
where $U_{0j} \sim \mathcal{N}(0, \tau_0^2)$ and independent over j and with respect to R_{ij}.

Linear mixed model: $(\texttt{science})_{ij} = \gamma_{00} + U_{0j} + R_{ij}$

Marginal model: $(\texttt{science})_{ij} \sim \mathcal{N}(\gamma_{00}, (\tau_0^2 + \sigma^2))$.

(d) Null/empty/baseline HLM — Same as (c)

(e) Random intercept with `math`

Hierarchical model :

Level 1: $(\texttt{science})_{ij} = \beta_{0j} + \beta_{1j}(\texttt{math})_{ij} + R_{ij}$
where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2:

$$\begin{aligned}\beta_{0j} &= \gamma_{00} + U_{0j} \\ \beta_{1j} &= \gamma_{10}\end{aligned}$$

where $U_{0j} \sim \mathcal{N}(0, \tau_0^2)$ and independent over j and with respect to R_{ij}.

Linear mixed model: $(\texttt{science})_{ij} = \gamma_{00} + \gamma_{10}(\texttt{math})_{ij} + U_{0j} + R_{ij}$

Marginal model: $(\texttt{science})_{ij} \sim \mathcal{N}((\gamma_{00} + \gamma_{10}(\texttt{math})_{ij}), (\tau_0^2 + \sigma^2))$.

(f) Random intercept with `OCmath`

Hierarchical model :

Level 1: $(\texttt{science})_{ij} = \beta_{0j} + \beta_{1j}(\texttt{OCmath})_{ij} + R_{ij}$
where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2:

$$\begin{aligned}\beta_{0j} &= \gamma_{00} + U_{0j} \\ \beta_{1j} &= \gamma_{10}\end{aligned}$$

where $U_{0j} \sim \mathcal{N}(0, \tau_0^2)$ and independent over j and with respect to R_{ij}.

Linear mixed model: $(\texttt{science})_{ij} = \gamma_{00} + \gamma_{10}(\texttt{OCmath})_{ij} + U_{0j} + R_{ij}$

Marginal model: $(\texttt{science})_{ij} \sim \mathcal{N}((\gamma_{00} + \gamma_{10}(\texttt{OCmath})_{ij}), (\tau_0^2 + \sigma^2))$.

(g) Random intercept with `grpCmath`

Hierarchical model :

Level 1: $(\texttt{science})_{ij} = \beta_{0j} + \beta_{1j}(\texttt{grpCmath})_{ij} + R_{ij}$
where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2:

$$\begin{aligned}\beta_{0j} &= \gamma_{00} + U_{0j} \\ \beta_{1j} &= \gamma_{10}\end{aligned}$$

where $U_{0j} \sim \mathcal{N}(0, \tau_0^2)$ and independent over j and with respect to R_{ij}.

Linear mixed model: $(\texttt{science})_{ij} = \gamma_{00} + \gamma_{10}(\texttt{grpCmath})_{ij} + U_{0j} + R_{ij}$

Marginal model: $(\texttt{science})_{ij} \sim \mathcal{N}((\gamma_{00} + \gamma_{10}(\texttt{grpCmath})_{ij}), (\tau_0^2 + \sigma^2))$.

(h) Random intercept with macro variable `grpMmath`

Hierarchical model :

Level 1: $(\texttt{science})_{ij} = \beta_{0j} + R_{ij}$
where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2: $\beta_{0j} = \gamma_{00} + \gamma_{01}(\texttt{grpMmath})_j + U_{0j}$
where $U_{0j} \sim \mathcal{N}(0, \tau_0^2)$ and independent over j and with respect to R_{ij}.

Linear mixed model: $(\texttt{science})_{ij} = \gamma_{00} + \gamma_{01}(\texttt{grpMmath})_j + U_{0j} + R_{ij}$

Marginal model: $(\texttt{science})_{ij} \sim \mathcal{N}((\gamma_{00} + \gamma_{01}(\texttt{grpMmath})_j), (\tau_0^2 + \sigma^2))$.

(i) Random intercept with micro variable `grpCmath` and macro variable `grpMmath`

Hierarchical model :

Level 1: $(\texttt{science})_{ij} = \beta_{0j} + \beta_{1j}(\texttt{grpCmath}) + R_{ij}$
where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.

Level 2:

$$\begin{aligned}
\beta_{0j} &= \gamma_{00} + \gamma_{01}(\texttt{grpMmath}))_j + U_{0j} \\
\beta_{1j} &= \gamma_{10}
\end{aligned}$$

where $U_{0j} \sim \mathcal{N}(0, \tau_0^2)$ and independent over j and with respect to R_{ij}.

Linear mixed model :

$$(\texttt{science})_{ij} = \gamma_{00} + \gamma_{01}(\texttt{grpMmath})_j + \gamma_{10}(\texttt{grpCmath})_{ij} + U_{0j} + R_{ij}$$

Marginal model :

$$(\texttt{science})_{ij} \sim \mathcal{N}((\gamma_{00} + \gamma_{01}(\texttt{grpMmath})_j) + \gamma_{10}(\texttt{grpCmath})_{ij}), (\tau_0^2 + \sigma^2)).$$

(j) Random intercept with micro variables `grpCmath`, `gender` and `grade`, and macro variable `grpMmath`

Hierarchical model :

Level 1:

$$(\texttt{science})_{ij} = \beta_{0j} + \beta_{1j}(\texttt{grpCmath}) + \beta_{2j}(\texttt{gender})_{ij} + \beta_{3j}(\texttt{grade})_{ij} + R_{ij}$$

where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent. Note $(\texttt{gender})_{ij} = 1$ if boy and $= 0$ if girl. The value of grade could be either $(\texttt{grade})_{ij} = 3$ or 4, or $(\texttt{grade})_{ij}$ could be dummy codes for grade level of the student.

Level 2:

$$\begin{aligned}
\beta_{0j} &= \gamma_{00} + \gamma_{01}(\texttt{grpMmath})_j + U_{0j} \\
\beta_{1j} &= \gamma_{10} \\
\beta_{2j} &= \gamma_{20} \\
\beta_{3j} &= \gamma_{30}
\end{aligned}$$

where $U_{0j} \sim \mathcal{N}(0, \tau_0^2)$ and independent over j and with respect to R_{ij}. All of the γ's are fixed in the population.

Linear mixed model :

$$\begin{aligned}
\texttt{(science)}_{ij} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} \\
&\quad + \gamma_{30}\texttt{(grade)}_{ij} + U_{0j} + R_{ij}
\end{aligned}$$

Marginal model :

$$\texttt{(science)}_{ij} \sim \mathcal{N}(\mu_{ij}, (\tau_0^2 + \sigma^2)).$$

where

$$\mu_{ij} = \gamma_{00}+\gamma_{01}\texttt{(grpMmath)}_j)+\gamma_{10}\texttt{(grpCmath)}_{ij}+\gamma_{20}\texttt{(gender)}_{ij}+\gamma_{30}\texttt{(grade)}_{ij}$$

Not Required but for the sake of completeness...

(k) Random intercept model with `grpCmath`, `grpMmath`, `grade`, and `gen_short` as explanatory variables.

Level 1:

$$\texttt{(science)}_{ij} = \beta_{0j} + \beta_{1j}\texttt{(grpCmath)} + \beta_{2j}\texttt{(gender)}_{ij} + \beta_{3j}\texttt{(grade)}_{ij} + R_{ij}$$

where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent. Note $\texttt{(gender)}_{ij} = 1$ if boy and $= 0$ if girl. The value of grade could be either $\texttt{(grade)}_{ij} = 3$ or 4, or $\texttt{(grade)}_{ij}$ could be dummy codes for grade level of the student.

Level 2:

$$\begin{aligned}
\beta_{0j} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(gen_short)}_{1j} + \gamma_{03}\texttt{(gen_short)}_{2j} \\
&\quad + \gamma_{04}\texttt{(gen_short)}_{3j} + U_{0j} \\
\beta_{1j} &= \gamma_{10} \\
\beta_{2j} &= \gamma_{20} \\
\beta_{3j} &= \gamma_{30}
\end{aligned}$$

where $U_{0j} \sim \mathcal{N}(0, \tau_0^2)$ and independent over j and with respect to R_{ij}. All of the γ's are fixed in the population.

Note:

$$\begin{aligned}
\texttt{(gen_short)}_{1j} &= 1 \quad \text{if “none” and 0 otherwise} \\
\texttt{(gen_short)}_{2j} &= 1 \quad \text{if “a little” and 0 otherwise} \\
\texttt{(gen_short)}_{3j} &= 1 \quad \text{if “some” and 0 otherwise}
\end{aligned}$$

Linear mixed model :

$$\begin{aligned}\texttt{(science)}_{ij} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(gen\_short)}_{1j} + \gamma_{03}\texttt{(gen\_short)}_{2j} \\ &\quad +\gamma_{04}\texttt{(gen\_short)}_{3j} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} \\ &\quad +\gamma_{30}\texttt{(grade)}_{ij} + U_{0j} + R_{ij}\end{aligned}$$

Marginal model :

$$\texttt{(science)}_{ij} \sim \mathcal{N}(\mu_{ij}, (\tau_0^2 + \sigma^2)).$$

where

$$\begin{aligned}\mu_{ij} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_j) + \gamma_{02}\texttt{(gen\_short)}_{1j} + \gamma_{03}\texttt{(gen\_short)}_{2j} \\ &\quad +\gamma_{04}\texttt{(gen\_short)}_{3j} + \gamma_{10}\texttt{(grpCmath)}_{ij} + \gamma_{20}\texttt{(gender)}_{ij} \\ &\quad +\gamma_{30}\texttt{(grade)}_{ij}\end{aligned}$$

(l) Random intercept model with `grpCmath`, `grpMmath`, `grade`, and `shortages` as explanatory variables.

Level 1:

$$\texttt{(science)}_{ij} = \beta_{0j} + \beta_{1j}\texttt{(grpCmath)} + \beta_{2j}\texttt{(gender)}_{ij} + \beta_{3j}\texttt{(grade)}_{ij} + R_{ij}$$

where $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent. Note $\texttt{(gender)}_{ij} = 1$ if boy and $= 0$ if girl. The value of grade could be either $\texttt{(grade)}_{ij} = 3$ or 4, or $\texttt{(grade)}_{ij}$ could be dummy codes for grade level of the student.

Level 2:

$$\begin{aligned}\beta_{0j} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(shortages)}_j + U_{0j} \\ \beta_{1j} &= \gamma_{10} \\ \beta_{2j} &= \gamma_{20} \\ \beta_{3j} &= \gamma_{30}\end{aligned}$$

where $U_{0j} \sim \mathcal{N}(0, \tau_0^2)$ and independent over j and with respect to R_{ij}. All of the γ's are fixed in the population.

Linear mixed model :

$$\begin{aligned}\texttt{(science)}_{ij} &= \gamma_{00} + \gamma_{01}\texttt{(grpMmath)}_j + \gamma_{02}\texttt{(shortages)} + j + \gamma_{10}\texttt{(grpCmath)}_{ij} \\ &\quad +\gamma_{20}\texttt{(gender)}_{ij} + \gamma_{30}\texttt{(grade)}_{ij} + U_{0j} + R_{ij}\end{aligned}$$

Marginal model :

$$\texttt{(science)}_{ij} \sim \mathcal{N}(\mu_{ij}, (\tau_0^2 + \sigma^2)).$$

where

$$\mu_{ij} = \gamma_{00}+\gamma_{01}\texttt{(grpMmath)}_j)+\gamma_{02}\texttt{(shortages)}_j+\gamma_{10}\texttt{(grpCmath)}_{ij}+\gamma_{20}\texttt{(gender)}_{ij}+\gamma_{30}\texttt{(grade)}_{ij}$$

2. Summary Table (10 points)

	number of		Fixed Effects			Random Effects				Fit statistics	
	estimated					Between		Within			
Model	parameters	"Effect"	γ	Estimate	SE	$\hat{\tau}^2$	SE	$\hat{\sigma}^2$	SE	-2 loglike	*AIC*
(a) Fixed Effects ANOVA	147		lots			n.a.		76.9537	1.2918	50964.1	51258.1
(b) ANCOVA	148		lots			n.a.		51.1241	0.8582	48061.8	48357.8
(c) Random Effects ANOVA	3	intercept		150.06	0.3955	20.9437	2.6867	78.5815	1.3331	51489.0	51495.0
(d) Null HLM	3	intercept		150.06	0.3955	20.9437	2.6867	78.5815	1.3331	51489.0	51495.0
(e) Random intercept	4	intercept	$\hat{\gamma}_{00}$	62.5902	1.4336	6.0149	0.8675	52.2296	0.8864	48482.2	48490.4
		`math`	$\hat{\gamma}_{10}$	0.5800	0.0094						
(f) Random intercept	4	intercept	$\hat{\gamma}_{00}$	150.22	0.2227	6.0149	.8675	52.2296	0.8864	48482.2	48490.4
		`OCmath`	$\hat{\gamma}_{10}$	0.5800	0.0094						
(g) Random intercept	4	intercept	$\hat{\gamma}_{00}$	150.04	0.3964	21.6691	2.6952	52.2039	0.8856	48646.8	48654.8
		`grpCmath`	$\hat{\gamma}_{10}$	0.5671	0.0096						
(h) Random intercept	4	intercept	$\hat{\gamma}_{00}$	14.0879	5.9217	3.0484	0.5719	78.6205	1.3340	51267.2	51275.2
		`grpMmath`	$\hat{\gamma}_{01}$	0.9016	0.3921						
(i) Random intercept	5	intercept	$\hat{\gamma}_{00}$	14.2280	5.9517	3.7105	0.5851	52.2304	0.8863	48426.4	48436.4
		`grpMmath`	$\hat{\gamma}_{01}$	0.9006	0.0394						
		`grpCmath`	$\hat{\gamma}_{10}$	0.5671	0.0096						
(j) Random intercept	7	intercept	$\hat{\gamma}_{00}$	10.3448	6.0164	3.7542	0.5884	51.7541	0.8782	48363.7	48377.7
		`grpMmath`	$\hat{\gamma}_{01}$	0.9002	0.0396						
		`grpCmath`	$\hat{\gamma}_{10}$	0.5528	0.0100						
		`grade`	$\hat{\gamma}_{20}$	0.9253	0.1953						
	`gender`	boys	$\hat{\gamma}_{30}$	1.1154	0.1720						
		girls		0	.						
(j) Random intercept	7	intercept	$\hat{\gamma}_{00}$	14.0460	5.9717	Everything else is the same					
w/ grade dummy	`grade`	3rd	$\hat{\gamma}_{20}$	-0.9253	0.1953						
coded		4th		0.0000	.						

Summary table (continued):

	number of estimated	Fixed Effects				Random Effects				Fit statistics	
						Between		Within			
Model	parameters	“Effect”	γ	Estimate	SE	$\hat{\tau}^2$	SE	$\hat{\sigma}^2$	SE	-2 loglike	AIC
(k) Random intercept	10	intercept	$\hat{\gamma}_{00}$	15.4795	6.0041	3.6278	0.5711	51.7470	0.8780	48359.0	**48379.0**
		`grpCmath`	$\hat{\gamma}_{10}$	0.5528	0.01001						
		`grpMmath`	$\hat{\gamma}_{01}$	0.8926	0.04014						
	`gender`	boy	$\hat{\gamma}_{20}$	1.1193	0.1720						
		girl		0	.						
	`grade`	3rd	$\hat{\gamma}_{20}$	-0.9233	0.1953						
		4th		0	.						
	gen$-$short	none	$\hat{\gamma}_{02}$	-0.04519	0.5859						
		a little	$\hat{\gamma}_{03}$	-0.9318	0.6303						
		some	$\hat{\gamma}_{0}$	0	.						
		a lot	$\hat{\gamma}_{04}$	0.03570	1.0732						
(l) Random intercept	8	intercept	$\hat{\gamma}_{00}$	11.0335	6.1988	3.7509	0.5878	51.7533	0.8782	48363.5	**48379.5**
		`grpCmath`	$\hat{\gamma}_{10}$	0.5528	0.01001						
		`grpMmath`	$\hat{\gamma}_{01}$	0.8961	0.04056						
	`gender`	boy	$\hat{\gamma}_{20}$	1.1156	0.1720						
		girl		0	.						
		`grade`	$\hat{\gamma}_{20}$	0.9261	0.1953						
		`shortages`	$\hat{\gamma}_{02}$	-0.1035	0.2254						

3. (1 point) There are $N = 146$ schools and a total $n_+ = 7097$ total observations.

4. (3 points) Yes, it appears that we need a random effects model. Evidence for this comes from the random effects ANOVA (model (c)) and the random intercept model with no explanatory variables (i.e., model (d)):

   - $\hat{\tau} = 20.9437$ is large relative to it's standard error, which equals 2.6867.
   - A rough 95% confidence interval for τ

$$\begin{aligned} \hat{\tau} &\pm 1.96(2.6867) \\ 20.9437 &\pm 5.2659 \longrightarrow (15.68, 26.21) \end{aligned}$$

   - (3 points) A measure of within group dependency, the intra-class correlation,

$$\rho_I = \frac{20.9437}{20.9437 + 78.5815} = \frac{20.9437}{99.5252} = .21$$

5. (3 points) Comparison of the fixed effects ANOVA, model (a), and the random effects ANOVA, model (c).

   Some things you could say

	(a) Fixed Effects ANOVA	(c) Random Effects ANOVA
Similarities	Within group variance assumption is the same; i.e., $R_{ij} \sim \mathcal{N}(0, \sigma^2)$ and independent.	
	The models are linear in the parameters.	
Differences	Interest is only in the selected schools	Interest in the population from which schools were sampled (schools are "exchangable")
	Only source of variance is within schools (σ^2)	Two sources of variance: within schools (σ^2) and between schools (τ_0^2)
	Estimate the effect of being in a particular school (using only that school's data), which is considered to be a fixed quantity	Estimate the variance (in the population) of the effects of being in a particular school using all the data at hand
	Lots of parameters (i.e., 146 fixed effects)	Only 1 parameter (τ_0^2)
	Fixed effects account for all the differences between schools	Effects between schools are random
	$\text{var}(Y) = \sigma^2$	$\text{var}(Y) = \tau^2 + \sigma^2$

6. (3 points) How are models (b) and (f) the same/different?

   They both include $\gamma\texttt{(OCmath)}_{ij}$ where γ is a fixed (the same for all schools and individuals), and their similarities and differences follow those listed above.

7. Comment on the relationship between models (c) and (d).

   They are the same.

8. Consider models (e), (f) and (g). Based on substantive, interpretational and statistical considerations, do you think student's math test scores should be centered around overall mean? Centered around the school mean? or Not centered at all? Why or why not?

   This question was put here to have you look at and think about centering using a particular data set (before we formally discuss it in class).

   Statistical Considerations: It appears that math as a micro level variable is needed in the model based

- On the decrease in σ^2 relative to the null model when math (i.e., $\texttt{(math)}_{ij}$, $\texttt{(OCmath)}_{ij}$ or $\texttt{(grpCmath)}_{ij}$) is included; that is, math helps to explain within groups variability in science scores.
- The value of $\hat{\gamma}_{10}$ for the math effect relative to it's standard error is "large".

There is no difference between models (e) and (f) in terms of their goodness of fit to the data; that is, raw math scores and overall mean centering are *statistically equivalent*. Model (g), which include group centered math scores, does not fits worse than models (e) and (f). This results from the fact that school differ on average in terms of their math scores, so the models with raw math scores and overall centered math scores lead to a decrease in both within and between group variance of the science scores.

Statistical considerations favor models (e) and (f).

Interpretational Considerations: Using overall mean centering, model (f), is preferable over raw math scores, model (f), because the intercept of model (f) can be interpreted as the science score of the average student's math score.

However, group mean centering, model (g), is preferable over either raw or overall centered math scores because interpretation of the "effect" of math on predicting science scores is problematic; that is, there are within group differences (which may result from one process) and between school difference (which may result from a totally different process). Thus, by using $\texttt{(grpMmath)}_{ij}$, this explains just the within school process…of course, we'ld also like to include $\texttt{(grpMmath)}_{j}$.

Substantive Considerations: This depends on whether we want to consider within and between group differences separately. I tend to think that in this case, where a student is relative to their peers is different from where a school is overall.

Overall: I favor (in this case), group mean centering (you don't have to agree with me, but your answer should be coherent and free from error.)

9. Consider models (d), (f), (g), (h) and (i)? Comment on the effect on the estimates of τ_o^2 and σ^2 of adding math scores in the model in different ways.

This question trys to get you to look at the different effect of adding micro and macro level variables.

Model	$\hat{\tau}_0^2$	$\hat{\sigma}^2$	Level of variable
(d)	20.94	78.58	Null model — no explanatory variables
(f)	6.01	52.23	Micro `(OCmath)`$_{ij}$, but schools differ systematically
(g)	21.67	52.20	Micro `(grpCmath)`$_{ij}$. Only a within schools
(h)	3.05	78.52	Macro only, `(grpMmath)`$_{ij}$
(i)	3.71	52.23	Macro and micro

When a micro variable that varies systematically over the schools, i.e., `(OCmath)`$_{ij}$, is added to the model, there is a decrease in both the variance estimates within and between groups (relative to the null model).

When a micro variable that does not systematically vary over schools is added to the null model, there is a decrease in only the within school unexplained variance of the science scores, i.e., σ^2.

When a macro variable is added to the null model, only the estimate of τ_0^2 decreases. The macro level variable helps to explain between school differences.

When both a micro level variable that does not vary systematically between schools and a macro level variable are included in the model, we get a decrease in the unexplained within and between group variances of the science scores. We can also say that the micro level variable (i.e., `(grpCmath)`$_{ij}$) is helping to explain within school variance and the macro level variable (i.e., `(grpMmath)`$_j$) is helping to explain the between school variance of the science scores.

10. Does it look like GENDER, GRADE and/or shortages of instructional supplies (entered either as a nominal discrete or as a numerical variable) are useful predictor/explanatory variables? Provide some rational for you answer.

    For this one you should look at $\hat{\gamma}$ relative to the standard error of the parameter. It is <u>very</u> difficult to look just at the change in σ^2 or τ_0^2 to decide whether a variable are useful.

    Answers for this question should shy away from statements including the word "significant" because we haven't talked about how to *statistically* test this.

    In looking at $\hat{\gamma}$'s versus their standard errors, the effects for gender and grade look "large" and therefore probably useful. The $\hat{\gamma}$'s for shortages, whether discrete or numerical versus their standard errors are "small" and probably are not useful.

11. Which is your favorite model (i.e, which do you think is the "best"). Why did you select this model? Interpret the results of this model.

Mine was (j). It has the smallest number of parameters relative to the goodness-of-fit of the model to data (note: the ANCOVA model actually fits better if we base our choice only on statistics).

You should then report the model and give an interpretation; that is, boys from school with high math scores who are also doing better on average than their peers in math have higher science scores,...

$$\widehat{\text{science}}_{ij} = 14.046 + .5528(\texttt{grpCmath})_{ij} + .9002(\texttt{grpMmath})_{j} + 1.1154\text{gender}_{ij} - .9253\text{grade}_{ij}$$

Boys tend to have science scores that are 1.12 points higher than girls; students that are 1 point higher than their peers in math tend to have science scores that are .55 points higher; and students in schools with 1 point higher mean math scores tend to have science scores that are .9 points higher (if you have a 10 point difference in math between schools, this would be 9 point difference in science scores).