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Overview/Summary of Multivariate Procedures: PCA = principal components analysis, CCA = canonical correlation analysis, MANOVA = multivariate analysis of variances, and DA = (linear) discriminant analysis.

	Multivariate Procedures			
Differences	PCA	CCA	MANOVA	(linear) DA
Data	1 set of $p \geq 2$ variables	2 sets: $p \geq 2$ variables and $q \geq 2$ variables	2 sets: group(s)/factors and (≥ 1) variables	2 sets: group/classification variable and $p \geq 2$ variables
Assumptions or requirements	– none –	$\boldsymbol{S}_{kk}$ (or $\boldsymbol{\Sigma}_{kk}$) are positive definite	$\boldsymbol{X} \sim \mathcal{N}_p(\boldsymbol{\mu}_k, \boldsymbol{\Sigma})$	equal $\boldsymbol{\Sigma}_k$
Focus on relationship	Within set of variables	Between sets of variables	Between groups relative to within groups	Between groups relative to within groups
Goals &/or Purposes	Account for as much variances as possible – interpretation – data reduction – diagnostics – input to other	Determine nature & strength of the relation between sets variables – how variables contribute to this	Statistical inferences regarding $\boldsymbol{\mu}_k$ (i.e., hypothesis tests regarding mean vectors)	(1) Discrimination or descriptions of differences between groups (eg, MANOVA) (2) Classification or prediction
Criterion	Variance of linear combination. Find $\boldsymbol{Y} = \boldsymbol{l}'\boldsymbol{X}$ that maximizes $\text{var}(\boldsymbol{Y}) = \frac{\boldsymbol{l}'\boldsymbol{S}\boldsymbol{l}}{\boldsymbol{l}'\boldsymbol{l}}$	Correlation between linear combinations of of variables of the two sets: maximize $\text{corr}(\boldsymbol{U}, \boldsymbol{U}) = \frac{\boldsymbol{a}'\boldsymbol{\Sigma}_{12}\boldsymbol{b}}{\sqrt{\boldsymbol{a}'\boldsymbol{\Sigma}_{11}\boldsymbol{a}}\sqrt{\boldsymbol{b}'\boldsymbol{\Sigma}_{22}\boldsymbol{b}}}$ where $\boldsymbol{U} = \boldsymbol{a}'\boldsymbol{X}$ and $\boldsymbol{V} = \boldsymbol{b}'\boldsymbol{X}$	Matrix needed $\boldsymbol{W}^{-1}\boldsymbol{B}$	Ratio of weighted squared distances between group & grand mean vectors relative to variance of (linear) discriminants. Find $\boldsymbol{y} = \boldsymbol{l}'\boldsymbol{X}$ that maximizes $\frac{\boldsymbol{l}'\boldsymbol{B}\boldsymbol{l}}{\boldsymbol{l}'\boldsymbol{W}\boldsymbol{l}}$

Similarities :

- All seek (or depend on) linear combinations of the original variables that maximize some criterion.
- The linear combinations are obtained from the eigenvalues of some matrix and the maximum value of the criterion equals the eigenvalue (or function of it) of the matrix.
- Successive linear combinations (given previous ones) can be found that maximize the criterion and are uncorrelated with the previous linear combinations.
- All techniques use the inter-relationship between variables (i.e., covariance or correlation matrix).
- All try to reduce the dimensionality of the problem (data reduction) and thus aid in the description and interpretation of relations between variables.
- Geometrically, all methods can be thought of as finding (or studying) sub-spaces of the original higher dimensional space.
- Expect for MANOVA and where statistical inferences (hypothesis testings, confidence statements) are desired, none of the techniques requires the assumption of multivariate normality.
- Except for PCA, all can be thought of as a special case of canonical correlation analysis.